

ON THE SYNCHRONIZATION OF CANDLE OSCILLATORS

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Abstract. In this study we will investigate an interesting collective behavior of candles. It has been observed that when several candles burn close to each other they form a common flame that exhibits oscillations in size and brightness. If two such oscillators burn together, they interact and the oscillations of the resultant system depend on the distance between them. The aim of this investigation, inspired by Problem 5 of the International Young Physicists Tournament in 2021, is to theoretically explain the phenomenon through overlapping of hot gas flows and radiation, as well as to check our understanding and measure additional parameters experimentally using advanced techniques, such as high speed schlieren photography.

Keywords: candle; flame; oscillator; schlieren photography; IYPT

1. Introduction

Even though we often use candles in our daily lives, we still do not completely understand all the complex physical phenomena associated with them. The phenomenon of interest in this article is one of them. It turns out that when two candles are placed at relatively small distances (of the order of several centimeters), their flames change their brightness and size periodically with a fixed phase difference of either 0 or π . This can be easily observed in non-laboratory conditions when two thin candles are put together. Various experiments have been carried out to investigate the phenomenon, such as in (Chen 2019) by Chen et al., but no consensus has been reached as to what is the qualitative theoretical explanation. In fact, the different proposed models conflict each other. For example, Dange et al. (Suraj Dange, 2019) considers buoyancy driven air vortices to be the explanation of our observations, while Kitahata et. al. (Kitahata 2009) suggests that the flames synchronize due to radiation and convective heat transfer. In this article we will suggest a different model, which is in a way a combination of those mentioned above.

2. Theoretical explanation

In this study we will consider two different mechanisms of synchronization of the oscillators – through the overlapping of hot gas flows above the candles and

through their interaction through electromagnetic radiation. Let us consider the first case. A commonly known fact, which can be easily proven using schlieren imaging, is that the hot gas products of the burning rise due to the buoyant force from the significantly colder (and therefore denser) surrounding air. This forms a flow similar to the one depicted in Figure 1. It can be seen that its diameter d is almost constant with height in the investigated range. Now, if we place two such oscillators at a center-to-center distance $r < d$, the gas flows would overlap (Figure 2). This would create a region of higher pressure, which exerts an outward force to the other parts of the flow. It is easy to see that in the region of interest (near the flame) this force has a vertical component, pointing downwards for both flames, acting against the buoyant force. Those two forces lead to a periodic change in the shape of the flames and cause the phenomenon. Moreover, because the interaction is the same for the two flames, they oscillate without a phase difference. This is the so-called *in-phase mode* of oscillation.



Figure 1. Gas flow above a candle¹⁾

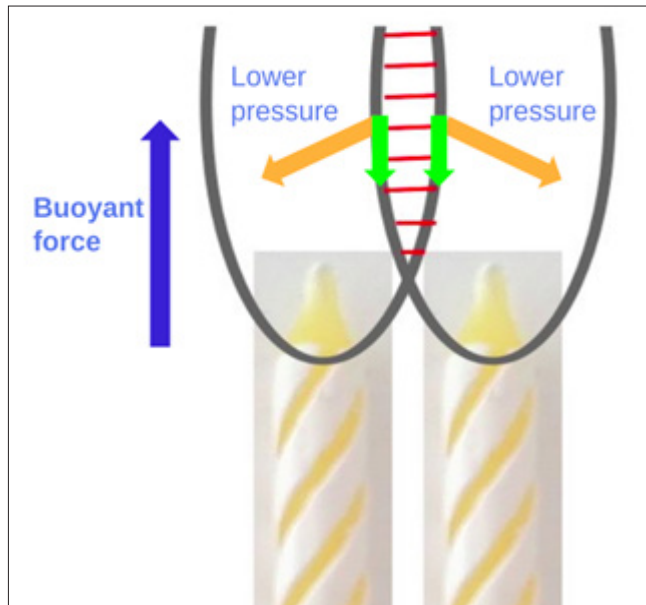


Figure 2. Synchronization mechanism in in-phase mode

Of course, it is possible that the oscillators are arranged so that $r > d$. Then the gas flows do not overlap and the effect we have described above does not occur. This gives us the ability to notice the action of a much weaker force — radiation pressure. The synchronisation principle is schematically shown in Figure 3. Let us assume that at the initial moment the flames of the two oscillators are with the same

height and do not oscillate. Then the two flames repel each other along a horizontal line connecting their centers. If due to a small perturbation one of the flames changes its height, the line of interaction is no longer horizontal and the total radiation force acting on each flame obtains a vertical component. As can be easily seen on Figure 3, those vertical components are pointing in exactly opposite directions, causing *anti-phase mode* synchronization.

To summarize, our model predicts that the oscillations of the system are either in-phase or anti-phase, i.e. the phase difference between the two flames is not a monotonous function of the distance as one could intuitively expect. Moreover, it predicts that there is some *critical distance*

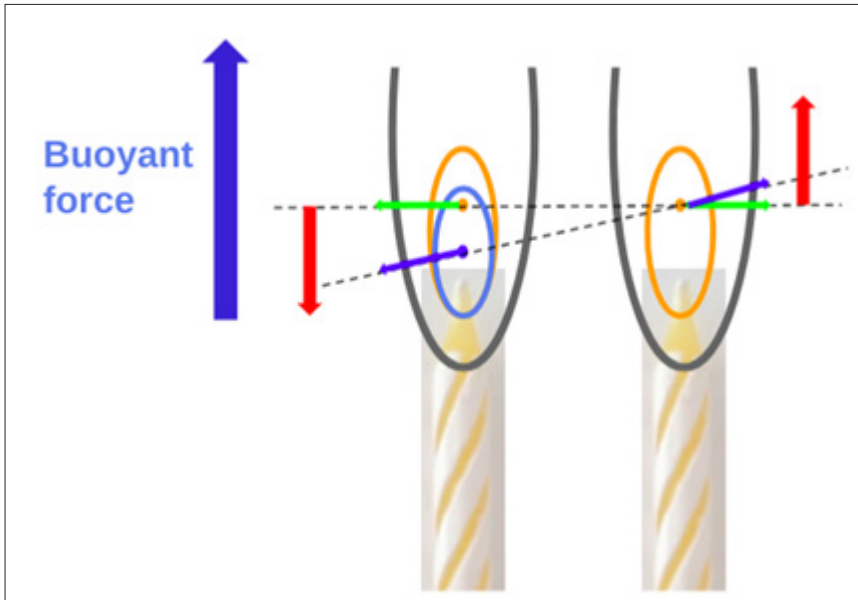


Figure 3. Synchronization mechanism in anti-phase mode

at which the mode changes and it is equal to

$$r_c = d,$$

where d is again the width of the air flow.

3. Experimental setup

3.1. High speed schlieren imaging

As already discussed, the phenomenon greatly depends on the properties of the hot gas flows forming around the flame. They are, unfortunately, invisible to the naked eye. For this reason, schlieren imaging was employed. A white LED, cov-

ered with a piece of aluminum foil with a small puncture on it, was used as an approximately point-like light source. It was positioned near the focus of a concave parabolic mirror ²⁾ (Figure 4). The emitted light gets reflected from the mirror and goes into a high speed camera³⁾ (Figure 5), also placed near the optical center. Since the hot gases have a different refractive index from air, the rays passing through the gas flow take a slightly different path and this creates the light and dark lines at its edges⁴⁾. The relative position of the components can be seen in Figure 6.

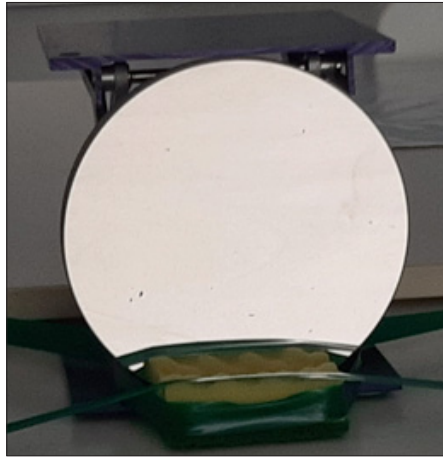


Figure 4. The mirror

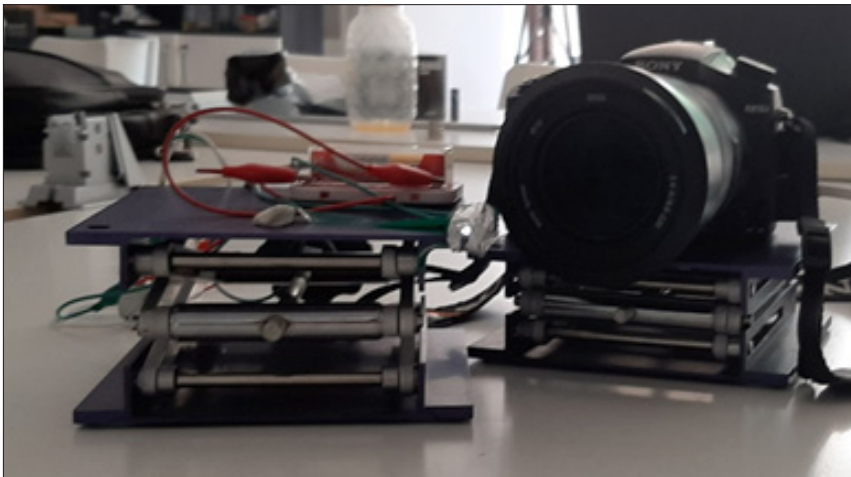


Figure 5. The light source and the camera



Figure 6. The schlieren setup

3.2. Oscillators

An oscillator in our case is any combination of one or more candles⁵⁾ that produces an oscillating flame. It is obvious that this definition is quite broad, so in this investigation we will not investigate all possible combinations, but only those in which there are two identical and symmetric oscillators, such as the ones you can see in Figure 7. Their main parameter is the number of candles N , which in our experiments takes the values 1, 2, 3, 4, 5 and 7.

Here it is also important to be said that the distance between the two oscillators was controlled by sticking them on a ruler using plasticine (Figure 8).



Figure 7. Oscillators used

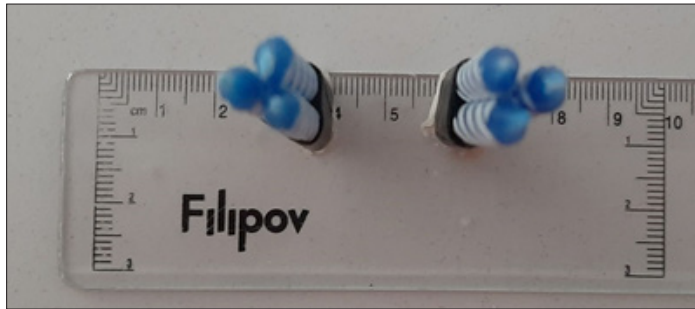


Figure 8. Oscillators' position

3.3. Setup for measuring frequency and amplitude

The setup already described is sufficient when the gas flows are investigated, but if we want to make more precise measurements it is necessary to prevent wind from disturbing the system. This is done using the setup depicted in Figure 9. The oscillators are put in a box with ventilation holes on it and the measurements are done by taking 240 fps ultra slow motion videos using a conventional smartphone⁶.

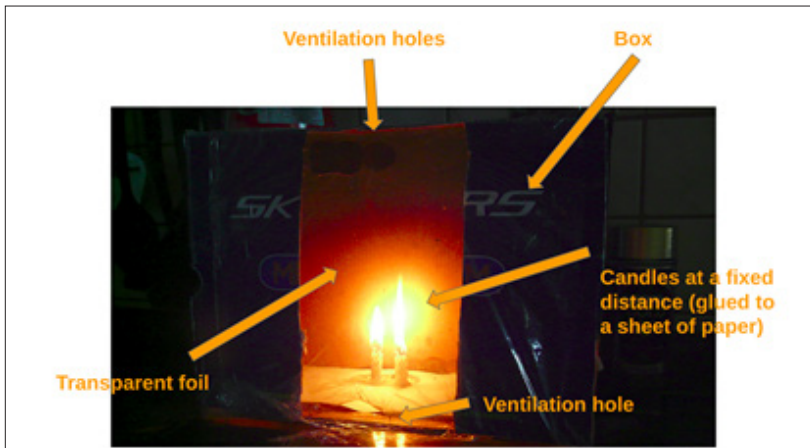


Figure 9. Second setup

4. Results

4.1. In-phase mode

It turns out that the gas flows around the flame take specific shapes throughout the oscillation, depending on the mode. Several stages of a single oscillation are shown by Figure 10⁷. It is also interesting to note that apart from the vertical oscillations, there are also horizontal ones and they are *anti-phase*.

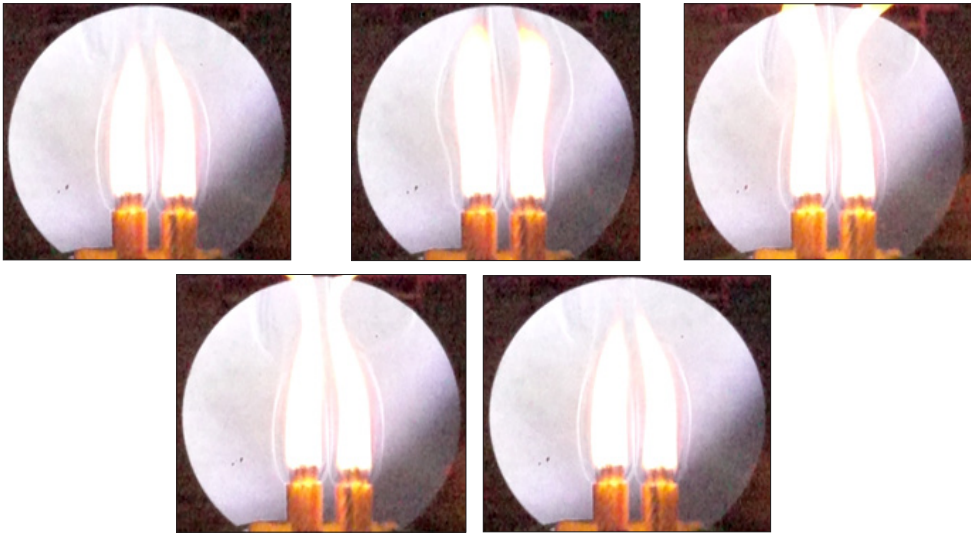


Figure 10. Different stages of an in-phase oscillation, $N = 7, r = 1.6cm$

4.2. Anti-phase oscillations

Again, the gas flows around the flames take a specific shape, which you can see in Figure 11. In this case the horizontal oscillations are *in-phase*.

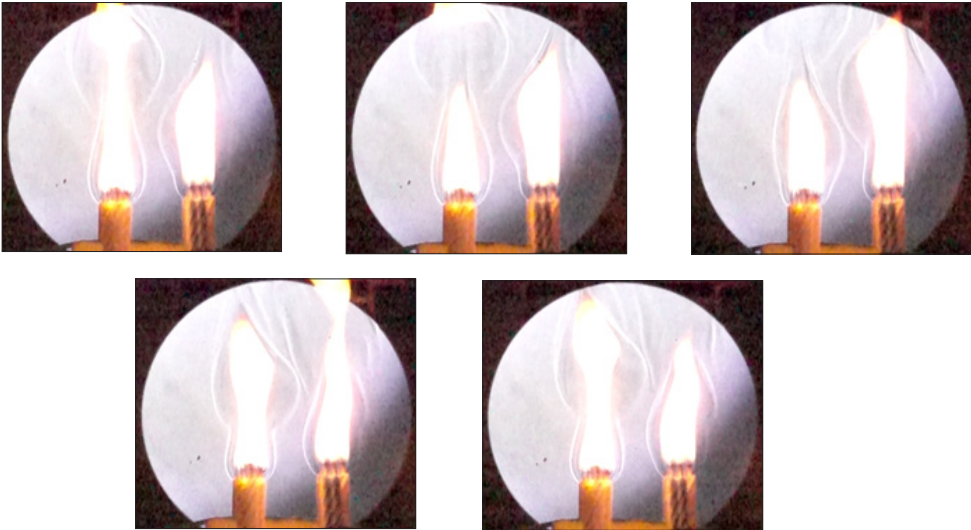


Figure 11. Different stages of an anti-phase oscillation, $N = 7, r = 3.6cm$

4.3. Flame puffing

It has already been observed that when one of the flames reaches its maximum (regardless of the mode) it might puff (Suraj Dange 2019). This was also observed in our experiments when the oscillations were strong enough. The phenomenon is demonstrated by Figure 12.

4.4. Critical distance

Our theoretical model predicted that the critical distance would be equal to the width of the gas flow. In order to check this, the resultant gas flow diameter was measured for all the oscillators, using the image editing software ImageJ¹³. In order to find the critical distance, high speed schlieren videos were taken for each combination of oscillators within the range 0.6 – 6.6 cm, with a step of 0.5 cm. Then the oscillations on the video were compared with the patterns presented in 4.1 and 4.2 in order to determine whether the oscillations are in-phase or anti-phase. This way r_c can be determined with an accuracy of ± 0.5 cm. The dependence of r_c and d on N is plotted on Figure 13 and the direct comparison between the two parameters is shown on Figure 14. It turns out that our theory fits the experimental data well.

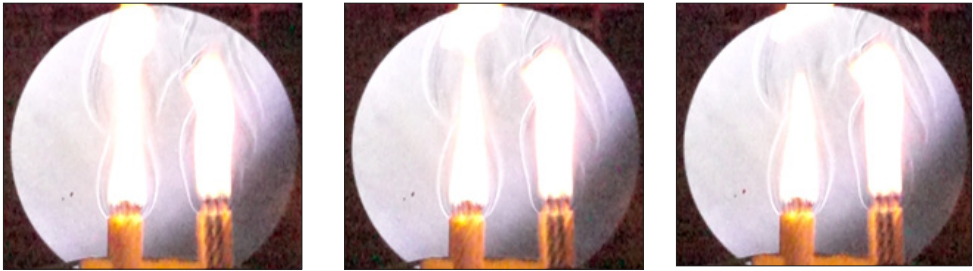


Figure 12. Different stages of flame puffing, $N = 7, r = 3.6$ cm

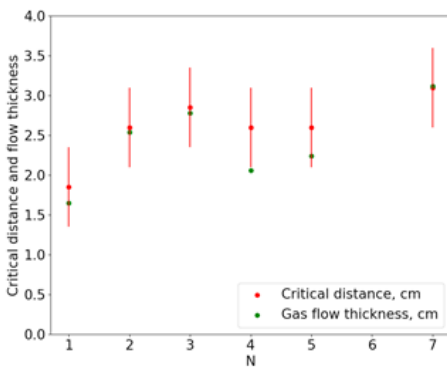


Figure 13. Critical distance and gas flow thickness as a function of N

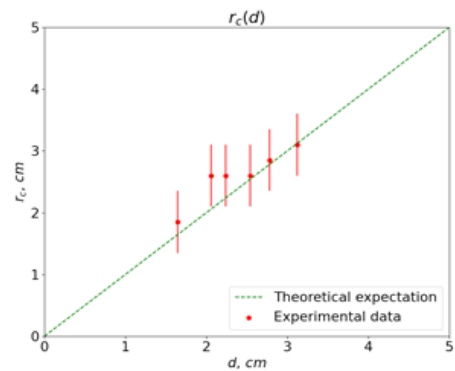


Figure 14. Comparison between r_c and d

4.5. Putting a barrier between the two oscillators

In our theoretical model we stated that in-phase synchronization is caused by gas flow overlapping, while anti-phase mode arises due to interaction through radiation pressure, without providing sufficient proof. We can partly do this by performing experiments, in which we put different barriers between the two oscillators. If we use a glass plate (or, in fact, any material that is transparent to electromagnetic waves, but blocks gas flow), we obtain an oscillation like the one depicted in Figure 15. It is important to note that the distance between oscillators has been selected so that normally there would be an in-phase synchronization. However, with the barrier the synchronization is anti-phase. This fits well with our theory – the glass lets through only the radiation, which, as we have already shown, induces anti-phase mode.

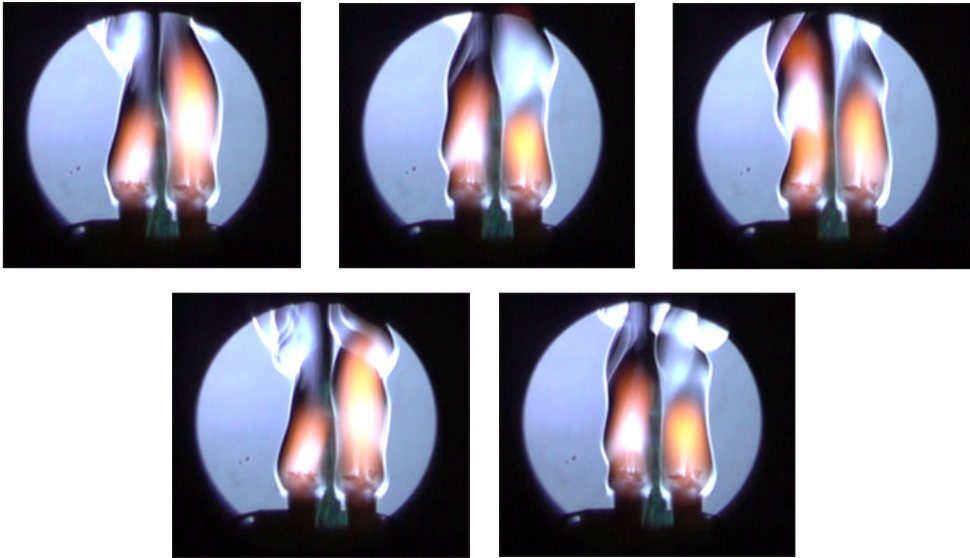


Figure 15. Oscillation cycle with a glass barrier, $N = 7, r = 2.1 \text{ cm}$

Another possibility is to use a piece of aluminum foil as a barrier – a material, which blocks the air flows and is opaque to electromagnetic waves, thus inhibiting both kinds of interaction considered by our model. Images from this experiment can be seen in Figure 16. No synchronization is observed in this case, which is consistent with our prediction that all interaction would be blocked.

4.6. Frequency and amplitude measurements

For these measurements the second setup was used. A series of experiments was conducted, in which the number of candles in each oscillator was fixed to $N = 3$

and r was varied. For each value of r were taken 10 measurements. Then, the y -coordinate⁸⁾ of each flame as a function of time was measured for each video using Tracker¹⁴⁾. Afterwards the extrema of $y(t)$ are identified for each case and from this the frequency and the amplitude are determined. Finally, in order to achieve better accuracy, the frequency at the given distance is taken to be the *average* over the 10 videos⁹⁾, whereas the amplitude is taken to be the *maximal* amplitude measured.



Figure 16. Experiment with aluminum foil barrier

There is also another effect that needs to be taken into account – due to small disturbances or experimental inaccuracies the two oscillators will have slightly different amplitude and frequency. We accept that the frequency of the system is the average frequency of the two oscillators and the amplitude is the bigger amplitude measured¹⁰⁾.

Now, let us move on to the data presentation – first for $y(t)$, which can be seen on Figure 17 for a case where flame puffing was observed and Figure 18 where it wasn't observed. An interesting feature of Figure 17 is the sudden change from minimum to maximum due to the separation of the flame.

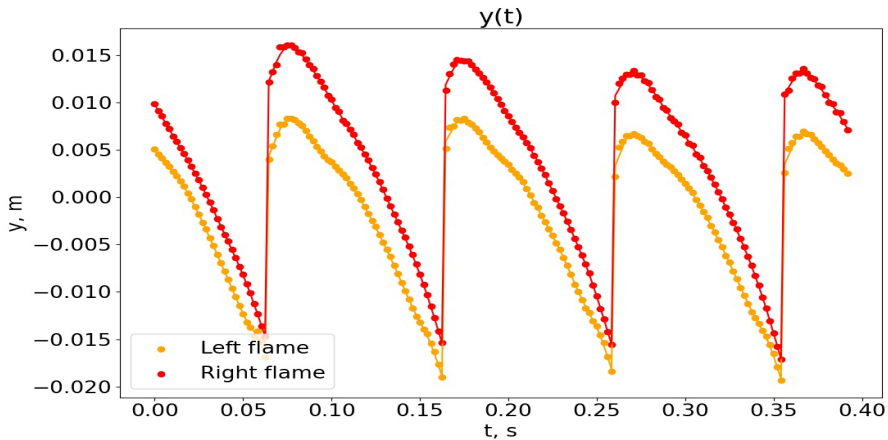


Figure 17. In-phase oscillation with flame puffing

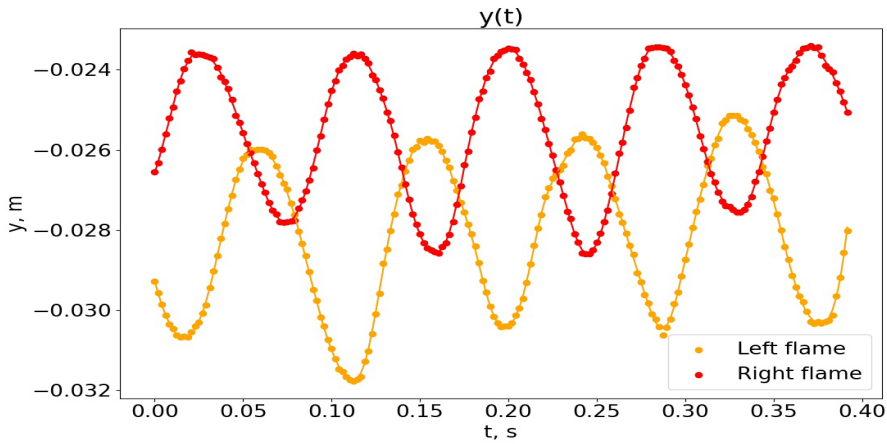


Figure 18. Anti-phase oscillation without flame puffing

Figures like 17 and 18 can also serve us to identify in-phase and anti-phase oscillations. In Figure 17 the maxima of the left flame¹¹⁾ happen at approximately the same time as the maxima of the right, which means that it demonstrates in-phase mode, whereas in Figure 18 the maxima of one oscillator roughly correspond to the minima of the other, meaning that the mode is anti-phase.

The next thing we will show is the dependence of the amplitude (Figure 19) and frequency (Figure 20) on the distance between the oscillators. By looking at the amplitude we can see that there is a sudden decrease of the amplitude near the critical distance¹²⁾, which is considered a separate mode by (Suraj Dange 2019) – *amplitude death mode*.

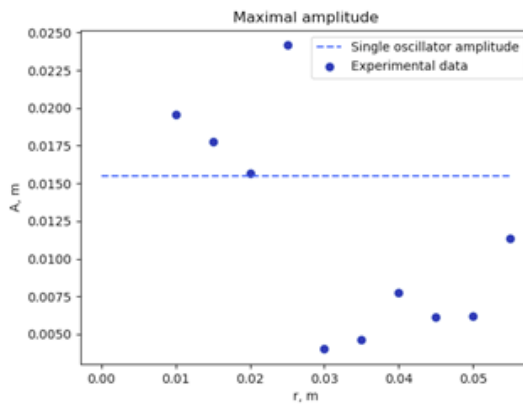


Figure 19. Amplitude variation

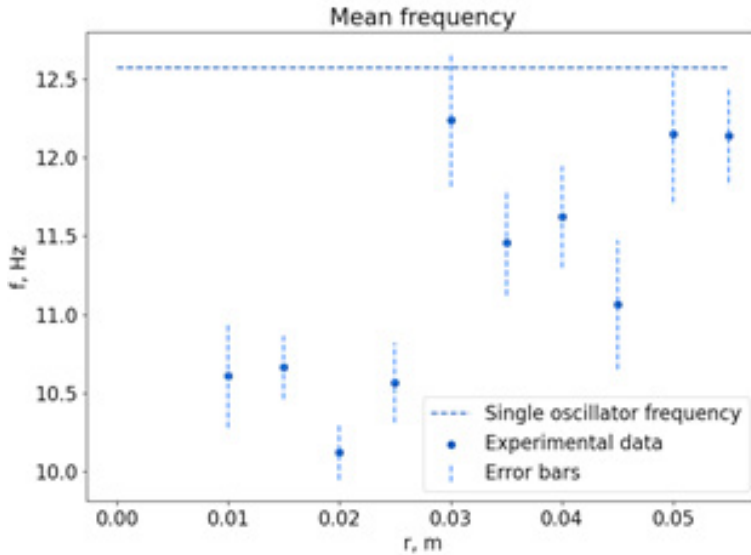


Figure 20. Frequency variation

The frequency is generally rising with the distance, but in a very narrow interval – from 10 to about 13 Hz. It is interesting to note that at large distances the amplitude and frequency seem to tend to the values specific to a single oscillator – $A = 1.5$ cm and $f = 12.6 \pm 0.8$ Hz. This is logical, given that the interaction between the oscillators is expected to be practically negligible at large distances.

4.7. Statistical approach to modes and the critical distance

By having a $y(t)$ comparison for 10 videos per distance we can measure the *probability* for each mode to occur at the given distance. This is done by observing the graphs for each video and determining whether the video is in-phase or anti-phase. Then the probability for each mode is the number of videos in which the mode was observed, divided by the total number of videos (10). The resulting relation is plotted on Figure 21. It can be easily seen that at $r < 3$ cm all oscillations are in-phase and at $r > 3$ cm they are mostly anti-phase. Near $r = 3$ cm both modes are equally dominant, so this should correspond to the critical distance – $r_c = 3.0 \pm 0.5$ cm. Within the margin of error, this corresponds to the value determined by observing the schlieren videos – 2.75 ± 0.50 cm.

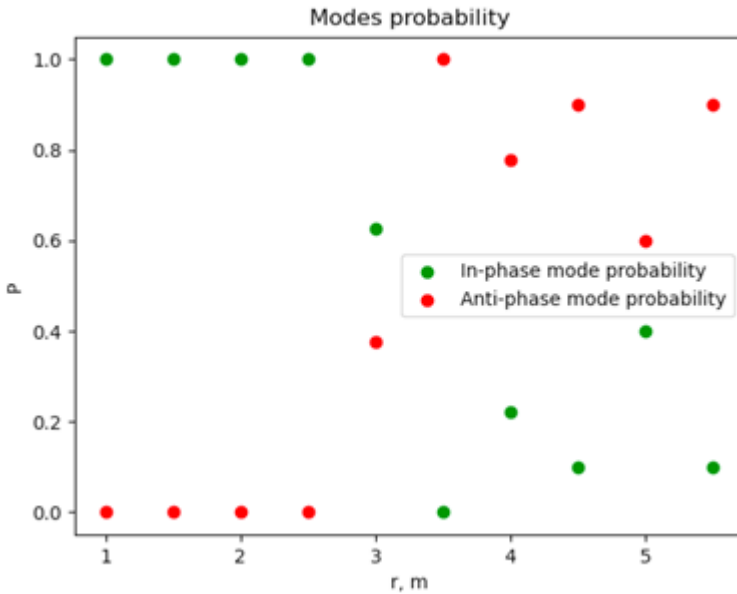


Figure 21. Mode probability as a function of distance

5. Conclusions

In this article a previously known, but vaguely explained phenomenon was investigated – the synchronisation of two candle flames. It turned out that it can be adequately described by interaction through gas flow overlapping and radiation pressure. From this theory followed the well-known fact that the synchronisation can be in either in-phase or anti-phase mode, depending on the distance. A high speed schlieren setup was constructed, with which the gas flows around the candles were investigated. It was found that their shapes throughout the oscillation vary in a way that is specific to the mode. Then, the critical distance was measured for different oscillators, falling in agreement with our model. An experiment in which barriers are introduced between the candles was performed, giving results that match our predictions. In order to quantify the oscillations, the frequency and amplitude were measured. The frequency appeared to increase with distance, while the amplitude exhibited a minimum.

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NOTES

1. The illustrations in this section are for oscillators, comprising only one candle. However, the conclusions drawn from them are equally valid for more candles.
2. Newtonian telescope mirror with a diameter of 150 mm and focal length of 750 mm.
3. In our experiments it was used at 1000 FPS.
4. The explanation provided here is brief and focuses on the specific setup. For more information it might be interesting for the reader to refer to [Davies, T. P., 1981]
5. Thin paraffin candles.
6. The high speed camera from the schlieren setup was not use because at the time of experiment conduction it was impossible to enter the laboratory it belongs to due to COVID-19 restrictions.
- 7.) The presented photos are snapshots from the high speed videos. For the videos themselves, please contact the author.
8. The y-axis is vertical and pointing downwards (as per Tracker default settings), with an arbitrary origin.
9. With an error equal to the standard deviation error of the 10 obtained frequencies.
10. This amplitude is chosen this way because it is very sensitive to external disturbances, which tend to reduce it. If we were to take the mean, it would be close to 0.
11. In order to distinguish them the flames of the two oscillators are called “left” and “right” according to their relative position in the setup and the resulting videos.
- 12.) According to our schlieren photography measurements it is 2.75 ± 0.50 cm, while the minimum of the graph is at 3.00 ± 0.50 cm.
13. <https://imagej.nih.gov/ij/>
14. <https://physlets.org/tracker/>

REFERENCES

- Chen, T., Guo, X., Jia, J. et al., 2019. Frequency and Phase Characteristics of Candle Flame Oscillation. *Sci Rep.* **9**, 342.
- Suraj Dange, Samadhan A. Pawar, Krishna Manoj & R. I. Sujith, 2019. Role of buoyancy-driven vortices in inducing different modes of coupled behaviour in candle-flame oscillators. *AIP Advances* **9**, 015119.

Kitahata, H., Taguchi, J., Nagayama, M., Sakurai, T., Ikura, Y., Osa, A., Sumino, Y., Tanaka, M., Yokoyama, E. & Miike, H., 2009. Oscillation and Synchronization in the Combustion of Candles. *The Journal of Physical Chemistry A*, **113**(29), 8164 – 8168.

Davies, T. P., 1981. Schlieren photography—short bibliography and review. *Optics & Laser Technology* **13**(1), 37 – 42.

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