https://doi.org/10.53656/math2025-3-7-tms

Educational Technologies
Образователни технологии

THE MAZE: SOLVING STRATEGIES OF PRIMARY SCHOOL STUDENTS

Alžběta Kolaříková, Radka Dofková
Palacký University Olomouc (Czech Republic)

Abstract. The maze as a didactic tool plays an important role in mathematics education because it provides a unique way to develop important skills. Solving mazes requires students to actively engage and apply various heuristic methods, which stimulates their creative thinking and ability to make effective decisions. This study focuses on the use of mazes and analyses the maze-solving strategies used by students aged 7 to 12 years to overcome them. Results from a study of 159 primary school students show that the most effective strategies are eliminating dead ends, simulating passage, and recording decisions. The study highlights the importance of promoting the development of students' own strategies and systematically recording their procedures as key elements for enriching mathematics education.

Keywords: heuristic methods; labyrinth; maze; primary school mathematics; solving strategies

1. Introduction

The definition of a maze from the earliest known paper discussing an algorithm for solving a maze is: "a maze is a continuous, convoluted path with insurmountable obstacles that enclose a space from which one or more exits can be made." (Wiener, 1873) This definition is more in line with the tasks used in the research, so we choose to use this term.

Mazes usually have an attractive appearance and children often encounter them already in kindergarten. Research suggests that perceptual and motor skills can be developed as early as in preschool children through this type of puzzle (Akpinar et al., 2021). In primary school, mazes are mainly used by younger students, but with slight adaptations, they are also beneficial for older age groups.

As research shows, the inclusion of mazes in lessons is often used by teachers (e.g., Hojjat et al., 2017). This is because they are not only a popular diversification of the lesson activities, but also easy to implement in the classroom, or useful for increasing students' motivation and engagement. Teaching based on maze solving also helps to develop key competences defined by the national educational framework, which is fully aligned with the currently prepared Czech curriculum document. In particular, maze solving directly addresses the problem-solving competency by fostering specific cognitive skills such as planning, spatial reasoning, and strategic decision-making.

Nowadays, many researchers use mazes in digital form and pursue teaching programming to children (e.g., Ternik et al., 2017; Zhang, Li, Lin et al., 2023). The purpose of this study is to highlight the usefulness of using mazes in mathematics education.

2. Theoretical Background

Mazes significantly contribute to the development of spatial reasoning, a key predictor of achievement in science and mathematics (Newcombe, 2010; Wai et al., 2009). This skill involves acquiring, organizing, and manipulating spatial knowledge (Arleo & Rondi-Reig, 2007), and its importance is reflected in its formal inclusion in many national curricula (Ramful et al., 2017). Beyond geometry, maze-solving fosters route planning, distance estimation, and 3D visualization (Davis, 2015; Martínez-Reyes & Hernández-Santana, 2012). Furthermore, it supports the development of fine and gross motor skills, which are particularly relevant for younger students in the context of writing acquisition (Akpinar et al., 2021; Cienfuegos et al., 2024).

From a cognitive perspective, solvers employ various strategies when navigating junctions. These range from egocentric (route-based) strategies, where students recall a sequence of directions, to allocentric (map-based) strategies, where they orient themselves using landmarks or internal mental representations (Iglói et al., 2009; O'Keefe & Nadel, 1978; Waller & Lippa, 2007). Moreover, integrating mazes into lessons has been shown to increase student motivation and cultivate positive attitudes towards mathematics (Antonova & Bontchev, 2019; Hojjat et al., 2017).

Various strategies facilitate maze navigation (Behrend, 2006). Well-known methods include the right-hand rule (following one side of the wall) and Ariadne's thread, which involves marking previously taken paths to avoid repetition (Lamb, 1979). Navigation essentially consists of a sequence of directional choices at junctions (Bock et al., 2024). This iterative decision-making process fosters the creation of mental maps and promotes logical problem-solving as students anticipate the spatial layout (O'Keefe & Nadel, 1978).

3. Method

3.1. Aim and Research questions

The aims of the research are as follows.

- To create a maze with different levels of difficulty suitable for primary school students.
- To analyse students' maze-solving strategies and compare the effectiveness of these strategies.
- To analyse the mistakes students made when solving the maze.

The following research questions were set at the beginning of the research investigation:

- RQ1: What specific strategies do primary students employ when successfully solving mazes, and how do these strategies reflect their problem-solving processes?
- RQ2: What types of errors occur during maze solving, and what do these errors reveal about students' spatial reasoning and planning?

3.2. Design and Sample

This study was conducted in the Czech Republic. For the research itself, a mixed design was chosen to exploit the potential of both approaches (Švaříček & Šeďová, 2007). The qualitative part of the research looked for the strategies used by the students in solving the maze and then used quantitative research methods to evaluate the extent to which these strategies were used.

In the Czech educational system, primary education (the first stage of basic education) comprises five grade levels (Grades 1–5), typically for children aged 6 to 11 or 12 years. The research sample included students from all five grades of one fully organized urban school, aged 7-12 years. The investigation took place in the school year 2022/2023 and involved 159 students, of whom 97 students were educated in a regular class and 62 students were educated in an honours class with extended mathematics instruction (hereafter referred to as the mathematics class). In total, 10 classes were included, representing the full primary education stage. A more detailed distribution of the research sample by grade is shown in Table 1.

Table 1. Frequency of respondents

Grade	Regular class	Mathematics class
1	17	14
2	20	14
3	22	14
4	16	10
5	22	10

In order to comply with the ethical parameters of the research, the consent of the school principal was first obtained to conduct the research, including the collection of student work.

To complement the analysis of written records, qualitative data collection was systematically integrated through post-task debriefings and semi-structured discussions with the students. These sessions allowed researchers to clarify the students' reasoning, identify their prior experiences with similar tasks, and understand the factors influencing their decision-making processes.

The data processing consisted of a first reduction of the collected data through summarizing and categorizing (Flick, 2018), while the second reduction involved the subsequent uncovering and interpretation of the meanings hidden in the students' solutions. For our purposes, the open coding method, as described by Švaříček and Šeďová (2007), was used. The coding process was conducted by the authors who analysed the students' written records, specifically the paths marked with pencils in the paper mazes, supplemented by notes from post-task discussions with the students.

To ensure the reliability of the analysis, the categories were cross-checked and discussed by researchers until a full consensus on the codes and the resulting system of categories was reached. The data were first divided into units. Each such unit was assigned a code that specified it and distinguished it from the others. Identical codes were assigned to the places of occurrence and then systematically categorized. A system of categories was created for each grade group separately, and then the relationships between the students' ages (grade groups) and tasks were observed.

3.3. Instrument

The main research tool was a maze task from a didactic test. The mazes that were presented to the students were graded using so-called conditions. The conditions in a maze are the elements that affect the passage through the path. Grade 1 students had a simple maze with no constraints. The passage condition for the Grade 2 maze (Fig. 1 left) was an even result of the numerical problem (hereafter referred to as the maze with the numerical problem). The upper grade students solved the maze with coloured gates (Fig. 1 right). The students in Grade 3 searched for a path that matched the given order of the coloured gates (maze with keys in order). The students in Grade 4 searched for the path that used the minimum number of red gates (maze with optimization problem). The students in Grade 5 were asked to solve the maze using the specified coloured gates but without a specified order (maze with unordered keys).

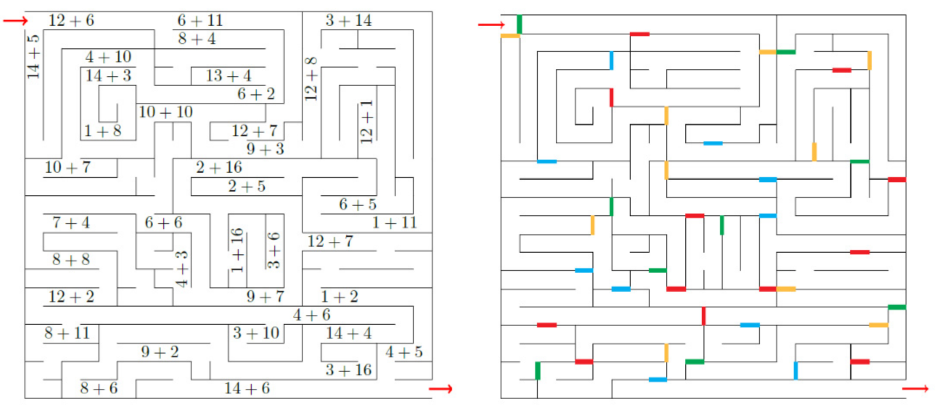

Figure 1. Sample assignment

4. Results

Students' strategies were identified by analysing the following areas:

- students' decision-making at junctions,
- the direction of the students' passage through the maze,
- actions performed before going through the maze,
- recording their own passage through the maze.

As part of the planned data collection, classroom discussions revealed that although all the students knew the principle of mazes and had solved them in the past, only 18% had encountered a maze that included passage conditions (based on a show of hands during the post-task debriefing). Nevertheless, 73% of all solvers found the correct path through the maze.

Figure 2 shows all identified strategies and their frequency and success rates. From the chart it can be seen that a 100% success rate was achieved by students who eliminated dead-ends, but at the same time this was the least used strategy (it was identified in only 6% of students). On the other hand, the most used strategy was finding the path from the start to the finish, which was used by 65% of the students.

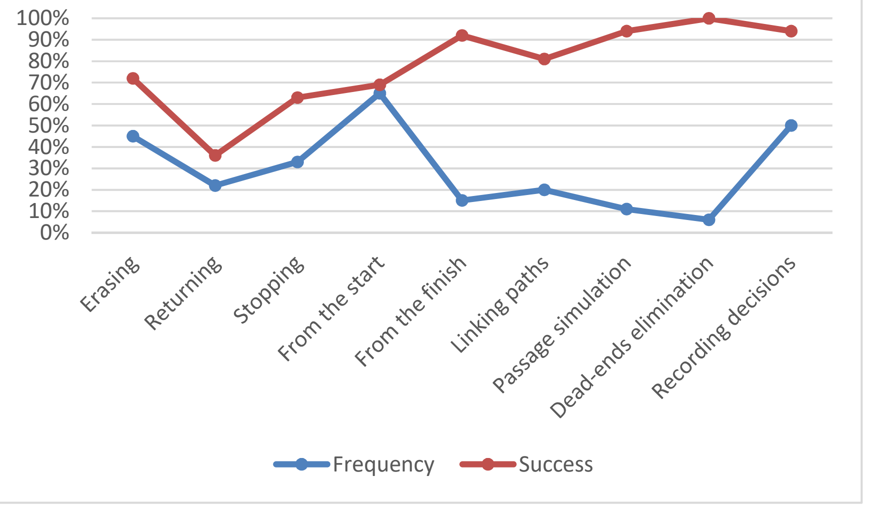

Figure 2. Frequency and success of strategies

Figure 3 shows the three strategies that students used to make decisions at junctions. The most used one was erasing. Students decided on a random path at a junction, and if they subsequently found that it did not lead to the finish, they deleted it or marked it as incorrect. Forty-five percent of students went through the maze in this way, of whom 72% were successful.

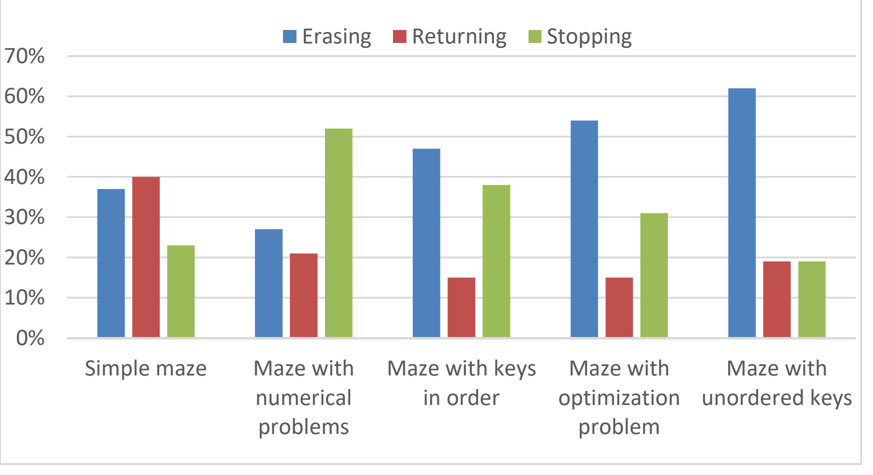

Figure 3. Frequency of strategies at junctions

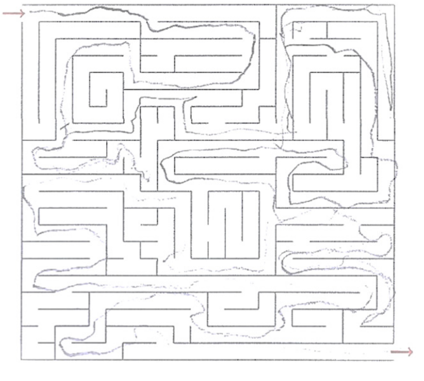

Figure 4. Returns at junctions

Conversely, the least used strategy was returning, which recorded the student's entire journey through the maze (Fig. 4). If the students wanted to change their original decision at the junction, they returned by the same way. This strategy was used by 22% of all students and of these, 36% were successful.

The last strategy observed at the junctions was stopping to allow students to view the area around the junction with possible paths, and only then plotting the verified route in the maze. Nineteen percent of students stopped at junctions and 63% found the correct route.

Three types of strategies were also identified for the maze direction area, as shown in Figure 5. Students most often solved the maze in the direction from the start to the finish. In total, 65% of students chose this direction. However, the success rate of this strategy was the lowest, with 69% of the students who chose this strategy going through the maze correctly. On the other hand, the most successful were the students who chose the opposite direction, that is, from the finish to the start. This strategy had a 92% success rate and was used by only 15% of students. The last strategy combined the two previous strategies and was linking paths. Students alternated through the maze from the start and from the finish until their paths intersected. The paths were connected by 20% of the students and 81% of them successfully solved the maze.

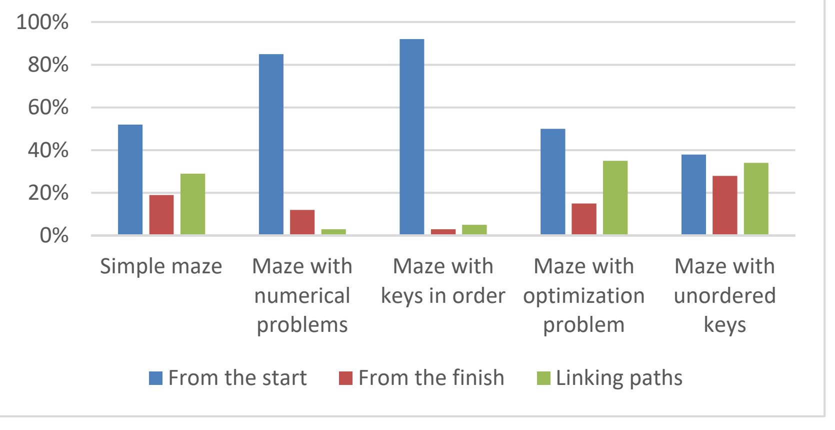

Figure 5. Frequency of direction strategies

The next strategies were identified as related to the actions performed before going through the maze. The simulation of the passage was most frequently used by the youngest students who solved the simple maze. They first traced the maze with their fingers, and then they recorded the path through the maze. In total, 11% of the students used this strategy and 94% of them solved the maze correctly. The dead-end elimination strategy had a 100% success rate. It consisted of identifying paths that did not lead to the finish and marking these paths before actually going through the maze (Fig. 6). This strategy was used by 6% of the students.

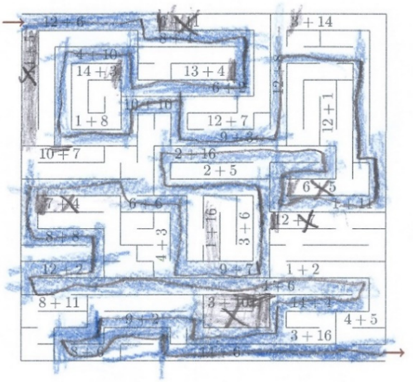

Figure 6. Dead-ends elimination strategy

Students who recorded their decisions also had a high success rate. However, this strategy could only be identified in students who solved the mazes with the coloured keys. For these tasks, it was observed whether students marked the keys they had just used when going through the maze. The so-called keys were represented in the task by stars and could be used just once to pass through a gate of a given colour. The keys were marked by 50% of the students and 94% of them found the correct path.

Students from regular classes erased most often at junctions (53%), while students from mathematics classes preferred the stopping strategy (44%). Figure 7 shows that in both types of classes, students used the returning strategy the least at junctions.

Students in both types of classes most often chose the passage from the start. In the regular classes, 70% of students solved the maze in this direction, while students in the mathematics classes used this direction in 56% of cases, and students in the mathematics classes were more likely to prefer the linking paths strategy compared to the other type of class. Other findings show an interesting difference between the use of the recording decisions strategy, which was used by 30% of students from regular classes but 88% of students from mathematics classes.

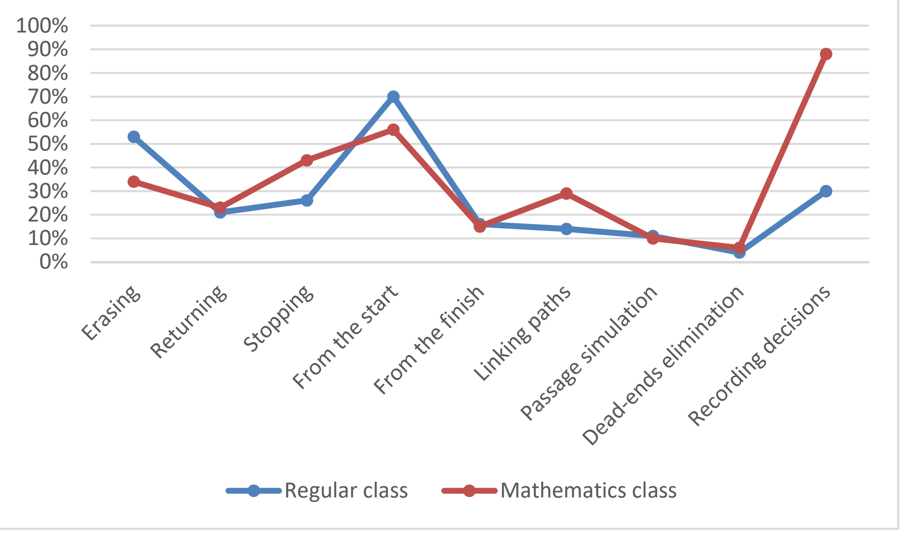

Figure 7. Distribution of strategies by type of class

A total of 69 girls and 90 boys participated in the study, with 68% of girls and 56% of boys attending a regular class. Gender did not play a significant role in the choice of strategies. The largest (though also rather marginal) difference was in recording decisions, which were made by 56% of boys but only 41% of girls. However, major differences can be observed in the choice of strategy at junctions. While girls were more likely to erase (52%), boys preferred returning (27%).

In total, three types of errors were identified. The most frequent error was following the path through the lines that represent the maze walls. Ten percent of the whole set of solvers went through the wall. When comparing the class types, this error was twice as frequent in regular classes (12%) as in mathematics classes (6%). As can be seen from Figure 8, the most frequent such errors were made by students who solved a simple maze (19%). Eight percent of students did not reach the finish line; they started the maze correctly, but during the task, they finished the passage without reaching the end. This error was more common in regular classes (11%) compared to mathematics classes (3%) and occurred most frequently among students who solved the maze with coloured keys.

Errors in the use of the wrong coloured gates were analysed only for the solutions of mazes with coloured gates. This error was most frequent for students who solved the maze with the optimization problem (27%). The correct solution for this maze was to pass through four red gates, which was achieved by 58% of the solvers. The second most common solution was the path with six red gates, which was found by 23% of the students.

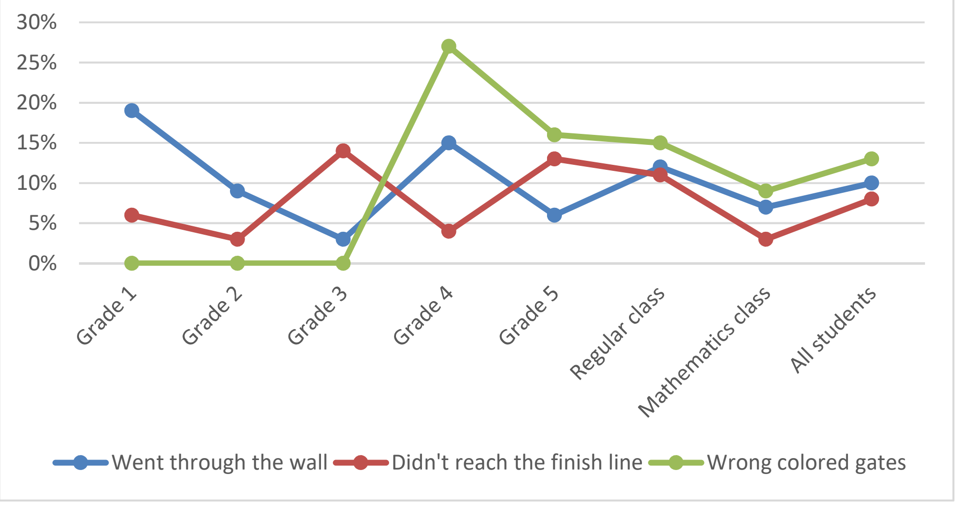

Figure 8. Frequency of mistakes

Qualitative data from the post-task discussions revealed specific reasons for some errors. One student, who failed to reach the finish line, explained that his prior experience with mazes was limited to those where the goal is located in the center, causing him to stop prematurely. Furthermore, observations and interviews indicated that several students who did not complete the journey showed signs of lower motivation and expressed a lack of interest in the task during the debriefing.

5. Discussion

The most successful strategy we have observed among students, the linking paths strategy, can be compared to the beacon strategy (Waller & Lippa, 2007), where travellers search for nearby, highly visible objects. In this strategy, students also moved towards their destination sequentially by strategically dividing the route into smaller sections. Similarly, students tried to aim for visible waypoints, i.e., places where one of their route segments ended. In this way, students naturally created landmarks that not only support orientation but also route learning.

The most frequently observed strategy was trial and error, which is common in problem solving where there is no clear-cut way to proceed. This strategy promotes students' patience, persistence, and ability to learn from their own mistakes (Schoenfeld, 1982; Young, 2009). Within heuristic strategies, students also intuitively used dead-end elimination and recording decisions without being taught these techniques by the teacher beforehand. This phenomenon is also observed in the study by Eisenmann et al. (2017), who point out the natural use of heuristic methods by students without prior explicit teaching of these methods.

Heuristic strategies are a key element in the problem-solving process, as they help students to recognize relationships between concepts and make more complex connections, develop more complex cognitive structures, increase students' knowledge, and can promote students' confidence in solving new problems (Lorenzo, 2005). Therefore, in the future, mathematics classroom instruction should aim to develop these strategies in students, and mazes can be one of the appropriate tools. Regular practice of heuristic methods, such as dead-end elimination or passage simulation, could promote better performance in mathematical tasks and in the ability to solve complex problems.

Students who went through the wall mostly answered that they did not notice the wall. While our study did not formally diagnose these factors, such errors may be associated with attention disorders, visual impairments, or temporary fatigue. The impairment of children's ability to solve mazes when tired is also mentioned by Gould & Perrin (1916). If students do not understand the principle of the maze, it may be appropriate to visit a real maze. In addition, real mazes have other benefits such as the possibility of demonstrating natural phenomena (e.g., Hojjat et al., 2017). Another option is to model the maze. However, even in real mazes, people sometimes do not follow the rules; in mazes made of bushes, one often encounters passages where the wall was originally. Vejmola (1991) points out that this rulebreaking is more often encountered in mazes with walls than in mazes whose paths are merely marked, although it is much easier to do so in the latter.

One student did not reach the goal because, as reported in the results, he relied on a specific mental model from previous experiences where the goal was centrally located. Similarly, the students who did not complete the journey were mostly those who exhibited less motivation. This is supported by research that has found that intrinsic learning motivation in younger school-age children is closely related to achievement and intelligence (Gottfried, 1990).

6. Limitation

The main limitation of the research is the relatively limited sample of students. The study focused on respondents from only one primary school, which may limit the generalizability of the results to a wider population. Another limitation is the lack of a deeper investigation into the influence of individual student characteristics, such as cognitive ability or motivation, which might affect their problem-solving strategies and success in navigating through the maze.

Another limitation lies in the chosen design of the maze tasks. Although tasks of varying difficulty were created, each student solved only one of them. The study could have included a wider range of tasks, for example with different mathematical concepts or multiple layers, to investigate how the different types of tasks would have influenced the students' strategies. Also, the timeframe of the research was limited, which may have had an impact on tracking the long-term development of skills and strategies that students might develop during regular encounters with mazes.

7. Conclusion

Based on the results of the research, the study suggests that the use of mazes in teaching mathematics at primary school offers valuable opportunities for the application of spatial imagination as well as for the engagement of logical thinking and problem-solving skills. Mazes function not only as a motivational element, but also as a means of practicing key skills, including navigation and planning. Research has shown that students use a variety of heuristic strategies, such as trial and error or "linking paths" strategies, which help them to better navigate through space and find optimal solutions. However, despite their potential benefits, mazes are rarely used in mathematics instruction, suggesting scope for more frequent integration of these tasks into school practice, particularly with an emphasis on systematic recording of progress and the development of heuristic methods.

We believe that a broader and more systematic involvement of mazes in mathematics education (not only at primary school, but also at other levels of education) has great didactic potential. Given the observed strategies indicating the involvement of spatial imagination and logical thinking, new types of maze problems could be developed and tested to include more advanced mathematical concepts such as geometry, algebra, or combinatorics. The plan could also be to integrate traditional mazes with digital technologies, such as interactive apps or virtual reality (Jeong et al., 2018), which would allow students to experience and practice mathematical and logical skills even more intensely.

By expanding the research to a larger number of schools and diverse settings, it would be possible to test how different demographic and geographic factors affect the effectiveness of mazes in the classroom. Research could also focus on long-term follow-up of students to better understand how the regular use of mazes in the classroom affects their overall educational outcomes.

Acknowledgments

The paper was written with the support of the grant IGA_PdF_2025_003 “Metacognitive Strategies in Solving Graph Theory Problems” at the Faculty of Education, Palacký University Olomouc. We have no conflicts of interest to disclose.

REFERENCES

Akpinar, S., Yontem, M. K., Talas, S., Hurmeric Altunsoz, I., Kilicarslan, A., & Akpınar, S. (2021). Effect of training program implemented with a maze-balance board on the developmental areas of preschool children. *Journal of Pedagogical Research.* 5(2), 49 – 60. http://dx.doi.org/10.33902/JPR.2021068625

Antonova, A., & Bontchev, B. (2019). Exploring puzzle-based learning for building effective and motivational maze video games for education. *EDULEARN19 Proceedings, IATED*, 2425 – 2434.

Arleo, A., & Rondi-Reig, L. (2007). Multimodal sensory integration and concurrent navigation strategies for spatial cognition in real and artificial organisms. *Journal of integrative neuroscience*, 6(03), 327 – 366. https://doi.org/10.1142/S0219635207001593

Behrend, M. (2006). How to solve a maze. *Caerdroia Journal*, 36, 10-17.

Bock, O., Huang, JY., Onur, O.A. et al. (2024). The structure of cognitive strategies for wayfinding decisions. *Psychological Research 88*, 476 – 486. https://doi.org/10.1007/s00426-023-01863-3

Cienfuegos, M., Maycock, J., Naceri, A., Düsterhus, T., Kõiva, R., Schack, T., & Ritter, H. (2024). Exploring motor skill acquisition in bimanual coordination: insights from navigating a novel maze task. *Scientific reports*, 14(1). https://doi.org/10.1038/s41598-024-69200-1

Davis, B. (2015). Spatial reasoning in the early years. *Taylor & Francis.* ISBN 978-1-315-76237-1.

Eisenmann, P., Přibyl, J., Novotná, J., Břehovský, J., & Cihlář, J. (2017). Volba řešitelských strategií v závislosti na věku [Choice of solution strategies as a function of age]. *Scientia in educatione, 8*(2), 21 – 38. https://doi.org/10.14712/18047106.432

Flick, U. (2018). *An Introduction to Qualitative Research.* 6th edition. Los Angeles: Sage. ISBN 9781526445650.

Gottfried, A. E. (1990). Academic intrinsic motivation in young elementary school children. *Journal of Educational psychology*, 82(3), 525. http://dx.doi.org/10.1037/0022-0663.82.3.525

Gould, M. C., & Perrin, F. A. C. (1916). A comparison of the factors involved in the maze learning of human adults and children. *Journal of Experimental Psychology*, 1(2), 122 – 154. https://doi.org/10.1037/h0072916

Hojjat, S., Fukuzaki, C., & Sowa, T. (2017). Maze and Mirror Game Design for Increasing Motivation in Studying Science in Elementary

School Students. In Interactivity, Game Creation, Design, Learning, and Innovation. 5th *International Conference, ArtsIT 2016, and First International Conference, DLI 2016, Esbjerg, Denmark,* May 2–3, 2016, Proceedings.

Iglói, K., Zaoui, M., Berthoz, A., & Rondi-Reig, L. (2009). Sequential egocentric strategy is acquired as early as allocentric strategy: Parallel acquisition of these two navigation strategies. *Hippocampus*, 19(12), 1199 – 1211. https://doi.org/10.1002/hipo.20595

Jeong, K., Lee, J., & Kim, J. (2018). A study on new virtual reality system in maze terrain. *International Journal of Human-Computer Interaction*, 34(2), 129 – 145. https://doi.org/10.1080/10447318.2017.1331535

Lamb, M. E. (1979). A Midsummer-Night's Dream: The Myth of Theseus and the Minotaur. *Texas Studies in Literature and Language*, 21(4), 478 – 491.

Lorenzo, M. (2005). The development, implementation, and evaluation of a problem-solving heuristic. *International Journal of Science and Mathematics Education*, 3, 33 – 58. https://doi.org/10.1007/s10763-004-8359-7

Martínez-Reyes, F., & Hern'ndez-Santana, I. (2012). The Virtual Maze: A game to promote social interaction between children. In 2012 *Eighth International Conference on Intelligent Environments*, *IEEE*, 331 – 334.

Newcombe, N. S. (2010). Picture this: Increasing math and science learning by improving spatial thinking. *American Educator*, 34, 29 – 35. https://doi.org/10.1037/a0016127

O'Keefe, J., & Nadel, L. (1978). The hippocampus as a cognitive map. *Clarendon Press*. ISBN 0-19-857206-9.

Ramful, A., Lowrie, T., & Logan, T. (2017). Measurement of spatial ability: Construction and validation of the spatial reasoning instrument for middle school students. *Journal of Psychoeducational Assessment*, 35(7), 709 – 727. https://doi.org/10.1177/0734282916659207

Schoenfeld, A. H. (1982). Expert and Novice Mathematical Problem Solving. *Final Project Report and Appendices BH.*

Švaříček, R., & Šeďová, K. (2007). *Kvalitativní výzkum v pedagogických vědách* [Qualitative research in educational sciences]. Prague: Portal. ISBN 9788073673130.

Ternik, Ž., Koron, A., Koron, T., & Šerbec, I. N. (2017). Learning programming concepts through maze game in Scratch. In *Proceedings at 11th European Conference on Games Based Learning*, 661 – 670.

Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for stem domains: aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology*, 101(4), 817 – 835. https://doi.org/10.1037/a0016127

Waller, D., & Lippa, Y. (2007). Landmarks as beacons and associative cues: Their role in route learning. *Memory & Cognition*, 35(5), 910 – 924. https://doi.org/10.3758/BF03193465

Wiener, C. (1873) "*Ueber eine Aufgabe aus der Geometria situs*" [About a problem from the Geometry situs] in Math. Annalen 6, 29 – 30.

Young, H. P. (2009). Learning by trial and error. *Games and economic behavior*, 65(2), 626 – 643. https://doi.org/10.1016/j.geb.2008.02.011

Zhang, M., Li, J., Lin, Y. et al. (2023). CoAR-Maze: empowering children's collaborative tangible programming in augmented reality. *CCF Trans. Pervasive Comp. Interact.* 5, 396 – 410. https://doi.org/10.1007/s42486-023-00135-8

✉ **Mgr. Alžběta Kolaříková**
ORCID iD: 0009-0006-1241-7382
Department of Mathematics,
Palacky University Olomouc,
Czech Republic
e-mail: alzbeta.kolarikova01@upol.cz

✉ **doc. PhDr. Radka Dofková, Ph.D.**
ORCID iD: 0000-0002-9568-6281
Department of Mathematics,
Palacky University Olomouc,
Czech Republic
e-mail: radka.dofkova@upol.cz