

THE IMPACT OF USING GEOGEBRA ON UNDERSTANDING QUADRATIC FUNCTIONS AND EQUATIONS FOR TENTH-GRADE STUDENTS

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Abstract. This study explores the impact of mathematical software, specifically GeoGebra, on tenth-grade students' understanding of quadratic equations and functions. The research was conducted in two classes with similar academic levels: one class (X/3) was taught using traditional methods, while the other (X/4) integrated mathematical software into the learning process. The objective was to analyze how these tools influence students' conceptual understanding, error reduction, and overall engagement in mathematics. The study identifies common difficulties and misconceptions students face while learning quadratic equations and functions. Various examples illustrate the errors encountered and highlight strategies to avoid them. The integration of mathematical software provided students with a more interactive and intuitive learning experience, significantly improving their problem-solving abilities. To assess the impact, a comparative analysis was performed using evaluation tests, questionnaires, and student interviews. The results revealed that students in class X/4 performed better in solving quadratic equations and graphing quadratic functions compared to their peers in class X/3. The Chi-square statistical analysis confirmed that the use of mathematical software positively influenced students' comprehension and accuracy in mathematical problem-solving. These findings emphasize the importance of incorporating technology in mathematics education to enhance conceptual understanding and engagement. The study suggests that educational institutions should integrate mathematical software into their curricula to foster a more effective learning environment. Future research can explore its long-term impact on mathematical proficiency and its application in other areas of mathematics.

Keywords: quadratic equations; quadratic functions; GeoGebra; student difficulties

1. Introduction and review literature

During the work with the students, we noticed that the motivation for them to learn mathematics in general was not great, and during this work, we tried to approach mathematics differently, especially for the subject of equations and

quadratic functions. During the development of the lessons, we tested two classes: in one, we explained equations and quadratic functions using the traditional method, while in the other, we taught the same topic through the use of mathematical software such as GeoGebra.

This paper aims to enhance student's success with equations and quadratic functions by integrating mathematical software tools such as GeoGebra, into classroom instruction. Additionally, we aim to boost students' motivation for learning mathematics, particularly in the context of equations and quadratic functions, through the use of mathematical software.

Mathematics, as a science, has practical applications in solving a wide range of everyday challenges. Equations and quadratic functions are a basic part of mathematics because, during the advancement in learning mathematics, the equations and quadratic functions will be an inseparable part of this process, so a good understanding of equations and quadratic functions is very important for students. After learning linear equations and functions from students, quadratic equations and functions are an advanced topic in mathematics, which falls under the more difficult type of equations and functions. Therefore, for easier understanding and concretization of exercises related to quadratic equations and functions, various mathematical software such as GeoGebra, etc., are used at this time. By using this mathematical software, the learning of this subject becomes very attractive, and the problems are understood very easily. The technology and the time we are living in make this easier because we can also use the resources from the internet for additional lessons. (Kamberi et. al 2022).

Knowing the great importance of quadratic equations and functions, many researchers have conducted studies on learning the quadratic equation and have pointed out that it is an important and interesting research object. There are many scientific works on this topic and each one has come to different conclusions depending on the research type. The primary focus of this study was to elicit a group of high school students' conceptions of quadratic equations with one unknown while considering concept definition and images as theoretical frameworks. The data initially showed that students could not provide a proper definition of quadratic equations with one unknown, and their definitions were not consistent with the formal (standard) definition of quadratic equations.

Students tried to define quadratic equations by stating some properties, which are valid for all equation concepts, instead of stating properties of quadratic equations. (Kabar 2018). The results indicate that most of the students used the factorization technique to solve quadratic equations. This result supports (Bossé & Nandakumar 2005), who claimed that a large percentage of the students preferred to apply the factorization techniques to find the solutions of quadratic equations. Also, in parallel with the results of (Bossé & Nandakumar 2005), the result of this study revealed that factoring the quadratic equations was challenging when they were presented to students in non-standard forms and structures (Didis & Erbas 2015). The results point to the need to create a new item on the research agenda for the international mathematics education research community: if quadratic equations are to remain an important component of lower and upper secondary mathematics curricula, then research is needed to guide teachers on how students think about quadratic equations, and especially on what can be done to help teachers improve students' understanding of variables in this context (Vaiyavutjamai et. al. 2005).

Students get confused when quadratic function concepts are presented in different ways they are not used to. The structure $y = ax^2 + bx + c$ (where $a \neq 0$ and a, b and c are constants) is the standard form of a quadratic function form revealing the location of the y -intercept $(0, c)$. The vertex form: $y = a(x - p)^2 + p$ distinctly highlights the turning point of the parabola (vertex) represented by $V(p, q)$. Lastly, the factored form: $y = a(x - x_1)(x - x_2)$ indicating the position of the x -intercept $(x_1; 0)$ and $(x_2; 0)$ (Mutambara et. al 2019). Based on the results and discussion of the research, the use of dynamic mathematical software as a learning tool for the topic of quadratic functions proves to be effective.

The media is deemed valid for classroom use based on ratings from media and material experts, as well as feedback from field practitioners and students. The software is superior to conventional teaching methods, as it allows students to effortlessly interpret the graphical representation of quadratic equations, enabling them to formulate broader generalizations. This automatically increases the student's learning achievement and enables math to be a more exciting subject (Barraza Castillo et. al 2014). This discovery provides evidence

that the application of GeoGebra can help students grasp the concepts of quadratic functions more effectively. Students of lower classes hardly find themselves when working with GeoGebra, compared to higher classes. (Mollakuqe et. al 2020). This tool offers a dynamic and engaging learning environment, assisting students in establishing connections between algebraic representations and their graphical interpretations. The GeoGebra application offers an intuitive way to present graphs of quadratic functions, enabling students to grasp their concepts more quickly and effectively. (Sumartini & Maryati 2021). The media is considered valid for classroom use based on ratings from media and material experts, as well as feedback from field practitioners and students. The software is superior to conventional teaching methods, as it allows students to easily interpret the graphical representation of quadratic equations and develop broader conceptual understandings (Barraza Castillo et. al 2014).

2. Using GeoGebra for understanding second-degree equations and quadratic functions

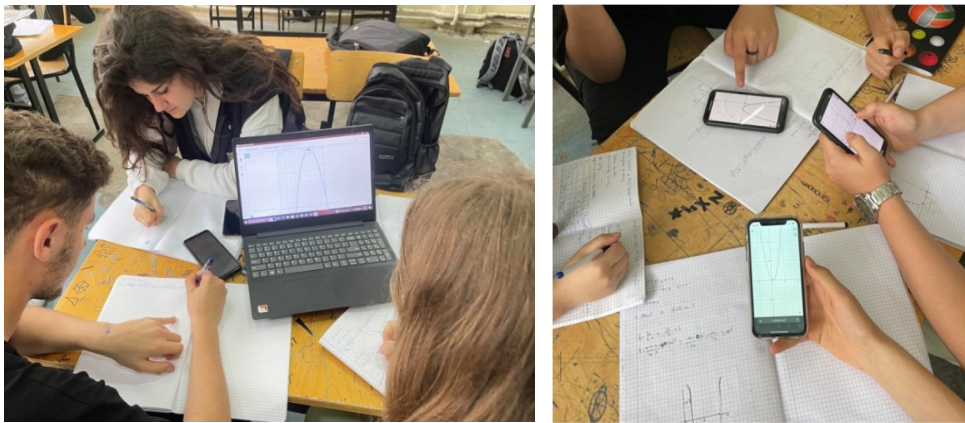


Figure 1. Students, while working with mathematical software

To improve and alleviate this difficulty among students, the researchers utilized mathematical software. The use of various math software tools significantly enhanced students' understanding of quadratic equations and

functions, while also making mathematics more engaging. This demonstrates that the software is more effective than traditional teaching methods, as it enables students to easily interpret and analyze the graphical representations of quadratic equations and functions (Wijaya et. al 2020). During the research with tenth-grade students, we utilized GeoGebra software. The integration of tools like GeoGebra further enhances the educational aspect, offering an interactive platform for students to visualize and understand exponential functions (Tuda et. al 2024). By enabling students to visualize and manipulate mathematical concepts, GeoGebra fosters deeper comprehension and greater motivation. However, for its successful implementation, adequate teacher training and equitable access to technological resources must be ensured (Aliu et. al 2025).

After we had traditionally explained quadratic equations and functions, the mistakes made by the students while solving the exercises were inevitable. The students mostly learned the formulas and the procedure of solving the exercises mechanically, and could not imagine how the equations of quadratic functions are represented in the graph.

3. The impact of using GeoGebra on the understanding of quadratic equations and quadratic functions

We conducted a study with two tenth-grade classes: in one, we used only the traditional method to explain equations and quadratic functions, while in the other, we incorporated mathematical software to teach the same unit. We then administered a test with identical exercises in both classes and obtained the following results. In Table 1 we marked 0 for no exercises solved, 1 for a solved exercise, 2 for two solved exercises, 3 for three solved exercises, 4 for four solved exercises, Class X/3 for students who learned with the classical method and Class X/4 for students who have learned through mathematical software. From the results we conclude that the class that used mathematical software during the learning achieved a better result in the test. Exercise II of the evaluation test was solved by a student in whose class the traditional teaching method was used, a mistake is seen in the exercise to find the peak of the function. Figure 2 shows where the student has mastered the formulas to find the peak of the

function. However, his confusion is in the graphical representation of the vertex by setting the vertex points as zeros of the function.

Table 1. The number of exercises solved during the assessment test

Number of test exercises	The number of students who have solved the exercises from class X/3	The number of students who have solved the exercises from class X/4
0	10	0
1	13	10
2	7	11
3	7	9
4	3	10

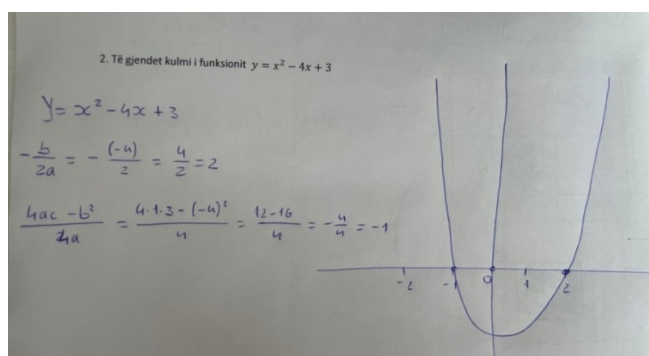


Figure 2. Exercise II of the test solved by the student from X/3

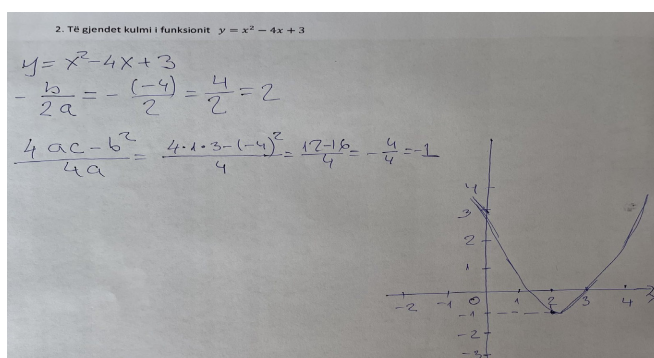


Figure 3. Exercise II of the test solved by the student from X/4

In Figure 3, the exercise was solved by an average student, but in whose class the modern method was used during the explanation of the quadratic equations and functions unit. It can be seen that the student has no problem at all with solving the exercise and presenting the function graphically.

In exercise III, the student was asked to choose a biquadratic equation. In Figure 4, the exercise was solved by a good student but in whose class the traditional method of explanation was used, and the student encountered difficulties in remembering the process of solving the exercise. The biquadratic equation is well replaced with $x^2 = t$, but then the student encountered difficulties while solving the created quadratic equation and did not come to the result.

3. Të zgjidhet ekuacioni bikuadratik $x^4 - x^2 - 2 = 0$

$$x^4 - x^2 - 2 = 0$$

$$x^2 = t \Rightarrow t^2 - t - 2 = 0$$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} = \frac{1 \pm \sqrt{1 + 8}}{2} = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2}$$

$$t = 2 \text{ or } t = -1$$

$$x = \pm \sqrt{2} \text{ or } x = \pm \sqrt{-1}$$

Figure 4. Exercise III of the test solved by the student from X/3

In Figure 5, the exercise was solved by a good student in whose class the modern method of explanation was used, namely for this problem the GeoGebra software was used, which shows a step-by-step solution to the exercise, and once the student has thoroughly mastered this type, they can successfully solve related exercises.

Now we consider exercise IV of the test. In Figure 6, we have the exercise solved by the student, a good student of the class where the traditional explanation method was used in that class, at first glance it can be seen that the peak of the function, the zeros of the function have been found exactly but their presentation on the graph is done completely wrongly. The student has confused the zeros of the function with its peak by placing them incorrectly on

the graph and has not been able to present the final form of the function since the points on the graph are irregularly placed.

3. Të zgjidhet ekuacioni bikuadratik $x^4 - x^2 - 2 = 0$

$$x^2 = t$$

$$t^2 - t - 2 = 0$$

$$t_{1/2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1}$$

$$t_{1/2} = \frac{1 \pm \sqrt{1+8}}{2}$$

$$t_{1/2} = \frac{1 \pm \sqrt{9}}{2}$$

$$t_{1/2} = \frac{1 \pm 3}{2}$$

$$t_1 = \frac{1+3}{2} = \frac{4}{2} = 2$$

$$t_2 = \frac{1-3}{2} = \frac{-2}{2} = -1$$

$$x^2 = t_1$$

$$x^2 = 2 \quad | \sqrt{}$$

$$\sqrt{x^2} = \sqrt{2}$$

$$x_{1/2} = \pm \sqrt{2}$$

$$x^2 = t_2$$

$$x^2 = -1 \quad | \sqrt{}$$

$$\sqrt{x^2} = \sqrt{-1}$$

$$x_{3/4} = \pm i$$

Figure 5. Exercise III of the test solved by the student of X/4

4. Të paraqitet grafikisht funksioni $y = x^2 + 6x + 8$

$$x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4 \cdot 1 \cdot 8}}{2 \cdot 1} = \frac{-6 \pm \sqrt{36 - 32}}{2}$$

$$= \frac{-6 \pm \sqrt{4}}{2} = \frac{-6 \pm 2}{2} \quad \begin{matrix} x_1 = -2 \\ x_2 = -4 \end{matrix}$$

$$-\frac{b}{2a} = \frac{6}{2} = 3$$

$$\frac{4ac - b^2}{4 \cdot a} = \frac{4 \cdot 1 \cdot 8 - 36}{4} = \frac{32 - 36}{4} = -\frac{4}{4} = -1$$

Figure 6. Exercise IV of the test solved by the student of X/3

In Figure 7, the exercise was solved by another good student in the class where the modern explanation method was used, that is, mathematical software was used and the student solved the exercise correctly and without mistakes. After the test results, we interviewed the student with the best success and the student with the weakest success.

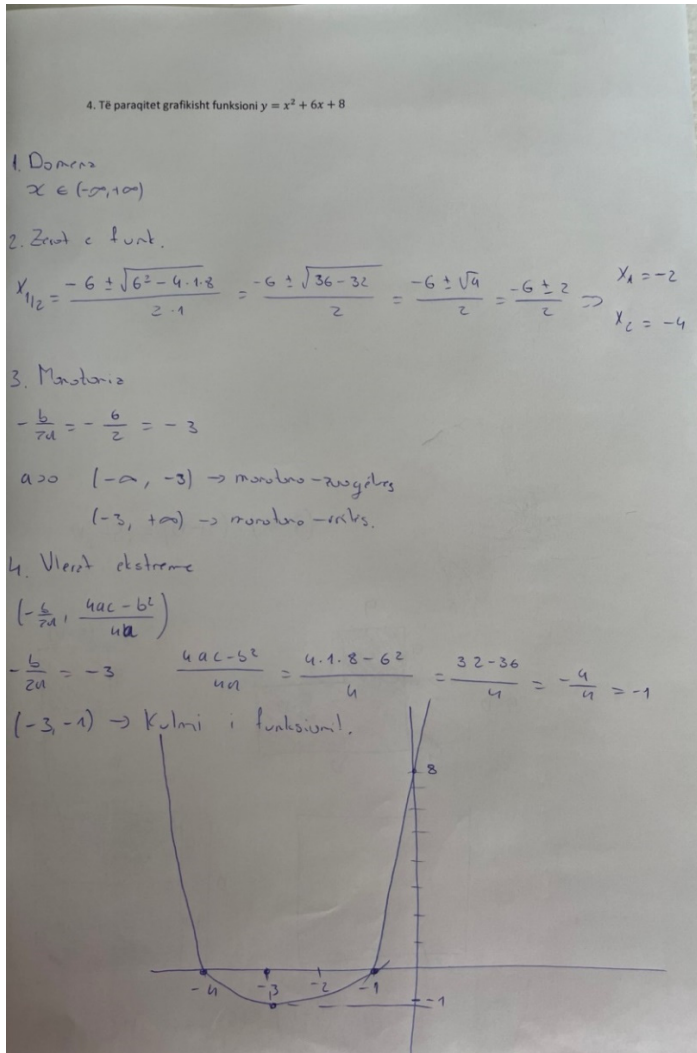


Figure 7. Exercise IV of the test solved by the student of X/4

Interview with the student with the best success

Interviewer: What was the problem you encountered while solving exercises with equations and quadratic functions?

Student: Acquiring formulas for solving quadratic equations, for finding the peak of a function, Viet's formulas, and other formulas.

Interviewer: Which math software do you use the most?

Student: GeoGebra

Interviewer: What math software has helped you the most when understanding quadratic equations and functions?

Student: They helped me concretize the exercise, especially on quadratic functions, that is, where the zeros of the function, the coordinates of the vertex and the shape of the graph of the function are placed.

Interview with the least successful student

Interviewer: What was the problem you encountered while solving exercises with equations and quadratic functions?

Student: I encountered difficulties in the algebraic part, i.e. in replacing numbers in formulas and then calculating them.

Interviewer: Which math software do you use the most?

Student: I don't use any of the software, as I'm not good at technology.

Interviewer: What math software has helped you the most when understanding quadratic equations and functions?

Student: I liked GeoGebra the most when you used it during the explanation in class because with it I saw the total selection of the exercise, i.e. step by step the entire solution process. But since I haven't practiced it myself to solve the exercises, I haven't been able to understand them well.

Students, in whose class the modern method of explanation is used, i.e., through mathematical thinking, think that the application of this method during

the explanation of mathematics, especially the equation and quadratic functions unit, is the best way for them to 'basically understand the problems and exercises in this learning unit. Through this approach, students believe that their focus on learning mathematical topics has significantly improved, as they are now more engaged with technology and its application in explaining mathematical concepts, particularly equations and quadratic functions. This, in turn, enhances classroom engagement and simplifies the learning process. Students, in whose class the modern method of explanation is used, i.e. through mathematical software such as GeoGebra, think that the application of this approach during the explanation of mathematics, especially the equation and quadratic functions unit, is the best way for them to 'basically understand the problems and exercises in this learning unit. Through this approach, they believe that the focus on learning mathematical topics has significantly increased, as they are now more passionate about technology and its application in explaining mathematical units, particularly equations and quadratic functions. This, in turn, enhances the classroom experience and simplifies the learning process. High school students in X classes find it easier to work with GeoGebra compared to students in lower grades (Aliu et. al 2021).

The difference between the students who used the modern method and the students who used the traditional method of learning is clearly visible, not only in the test results but also during engagement in class or even solving homework, since the students who used the modern method of learning, that is, with the application of the software, their engagement in class and the choice of homework without mistakes was significantly greater than among students where only the traditional method was used.

After both classes that participated in the research were introduced to the two methods of explanation, the traditional and the modern method, we created a questionnaire about which of the methods the students understood the learning unit more.

The questionnaire

1. What was the method by which you best understood quadratic equations and functions?

Next, we will calculate the value Chi-square:

$$X^2 = \frac{(31 - 28)^2}{28} + \frac{(2 - 4)^2}{4} + \frac{(3 - 6)^2}{6} + \frac{(4 - 2)^2}{2} = 4.82$$

With significance level $\alpha = 0.05$, we put $X^2_{1,0.05} = 3.841$, so the critical domain is $C = (3.841, \infty)$ and since $4.82 \in C$, we conclude that X and Y are dependent.

4. Discussion

This study aimed to explore the impact of using mathematical software, specifically GeoGebra, on tenth-grade students' understanding of quadratic equations and functions. The research was conducted in two classes with similar academic performance levels, where one class (X/3) was taught using traditional methods, while the other (X/4) integrated mathematical software into the learning process. The findings of this study indicate that the use of mathematical software significantly improved students' comprehension and problem-solving abilities related to quadratic equations and functions.

One of the key observations from this research was that students in class X/3, who followed traditional teaching methods, faced difficulties in visualizing quadratic functions and understanding the relationship between algebraic and graphical representations. Errors related to misidentifying coefficients, incorrect application of formulas, and challenges in solving quadratic equations were prevalent. Conversely, students in class X/4, who utilized mathematical software, demonstrated a higher level of accuracy in solving problems and a better conceptual understanding of quadratic equations and functions. The integration of technology enabled them to engage with interactive visualizations, making abstract concepts more tangible.

A crucial aspect of the research was the comparative analysis of test results between the two classes. The statistical evaluation revealed that students in class X/4 performed significantly better in solving quadratic equations and graphing quadratic functions. The use of Chi-square analysis confirmed that the success rate was higher among students who used mathematical software, highlighting its effectiveness in reducing common mistakes. This suggests that technological tools play an essential role in improving mathematical

comprehension by allowing students to explore and interact with mathematical concepts dynamically.

Additionally, the study examined the motivational impact of mathematical software on students. Interviews with students from both classes indicated that those in X/4 found the learning process more engaging and intuitive. The ability to visualize problems and receive instant feedback through software applications contributed to a deeper understanding and greater interest in mathematics. In contrast, students in X/3 often relied on memorization without fully grasping the underlying concepts, leading to confusion in problem-solving.

These findings align with existing literature that supports the integration of technology in mathematics education. Previous studies have shown that digital tools enhance students' ability to conceptualize and apply mathematical principles effectively. The results of this study reinforce the argument that incorporating technology into the curriculum can facilitate a more effective learning environment, particularly in topics that require graphical interpretation and algebraic manipulation.

Based on these findings, it is recommended that mathematics educators integrate software tools into teaching methodologies to enhance students' engagement and understanding. Schools should consider investing in technological resources and training teachers to use digital platforms effectively. Future research can explore the long-term impact of mathematical software on students' performance and its applicability to other mathematical topics.

In conclusion, this study highlights the transformative potential of mathematical software in learning quadratic equations and functions. By bridging the gap between algebraic expressions and their graphical representations, digital tools provide students with a more interactive and engaging learning experience. The improved results of students in class X/4 serve as strong evidence of the benefits of integrating technology in mathematics education, paving the way for more effective teaching strategies in the future.

5. Conclusion

This study demonstrated that the integration of mathematical software significantly enhances students' understanding of quadratic equations and

functions. Through a comparative analysis of two tenth-grade classes, it was observed that students who used software tools such as GeoGebra achieved higher results than those who followed traditional teaching methods. The findings confirmed that digital tools provide a more interactive, visual, and engaging approach to learning, leading to improved problem-solving skills and a deeper conceptual understanding of mathematical principles.

The results from the assessment test and Chi-square statistical analysis showed that students in class X/4 achieved higher accuracy rates and demonstrated fewer misconceptions compared to their peers in class X/3. The ability to visualize and manipulate equations graphically enabled students to correct errors more efficiently and understand abstract mathematical concepts with greater clarity. Furthermore, interviews with students indicated that the use of mathematical software increased motivation and interest in mathematics, making the learning experience more enjoyable and dynamic.

These findings emphasize the necessity of integrating technology into mathematics education. As digital tools become more accessible and user-friendly, educators should consider incorporating them into their teaching methodologies to support students' learning processes. This approach not only strengthens conceptual understanding but also prepares students for an increasingly technology-driven academic and professional landscape.

Given these findings, it is strongly recommended that educational institutions implement the use of mathematical software in their curricula. Future studies could investigate its impact on other mathematical topics and the long-term retention of mathematical concepts. Ultimately, embracing technology in education can transform the way mathematics is taught, making it more intuitive, effective, and engaging for students.

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REFERENCES

- ALIU, E.R., ZENKU, T.J., ISENI, E. AND REXHEPI, S., 2025. The Advantage of Using GeoGebra in the Understanding of Vectors and Comparison with the Classical Method. *International Electronic Journal of Mathematics Education*, 20(2).
- BARRAZA CASTILLO, R. I., CRUZ SÁNCHEZ, V.G., VERGARA VILLEGAS, O.O., 2015. A Pilot Study on the Use of Mobile Augmented Reality for Interactive Experimentation In *Quadratic Equations. Mathematical Problems in Engineering*, 2015(1).
- BOSSÉ, M. J., NANDAKUMAR, N. R., 2005. The Factorability of Quadratics: Motivation for More Techniques. *Teaching Mathematics and its Applications*, 24(4), pp.143-153.
- DIDIS, M. G., ERBAS, A. K., 2015. Performance and Difficulties of Students in Formulating and Solving Quadratic Equations with One Unknown. *Educational Sciences: Theory and Practice*, 15(4), pp.1137-1150.
- KABAR, M. G. D., 2018. Secondary School Students' Conception of Quadratic Equations with One Unknown. *International Journal for Mathematics Teaching and Learning*, 19(1), pp.112-129.
- KAMBERI, S., LATIFI, I., REXHEPI, S., ISENI, E., 2022. The Influence of Practical Illustrations on The Meaning and Operation of Fractions in Sixth Grade Students, Kosovo-Curricula. *International Electronic Journal of Mathematics Education*, 17(4).
- MOLLAKUQE, V., REXHEPI, S., ISENI, E., 2020. Incorporating Geogebra Into Teaching Circle Properties at High School Level and Its Comparison with The Classical Method of Teaching. *International. Electronic Journal of Mathematics Education*, 16(1).
- MUTAMBARA, L. H. N., TENDERE, J., CHAGWIZA, C. J., 2019. Exploring the Conceptual Understanding of the Quadratic Function Concept in Teachers' Colleges in Zimbabwe. *Eurasia Journal of Mathematics, Science and Technology Education*, 16(2).
- SUMARTINI, T. S., MARYATI, I., 2021. Geogebra Application for Quadratic Functions. In *Journal of Physics: Conference Series* (Vol. 1869, No. 1, p. 012138). IOP Publishing.

- TUDA, S., REXHEPI, S., 2024. Geogebra Impact in Avoiding Common Mistakes Students Make in Handling Exponential Functions. *Mathematics & Informatics*, 67(4).
- VAIYAVUTJAMAI, P., ELLERTON, N. F., CLEMENTS, M. A., 2005. Students' Attempts to Solve Two Elementary Quadratic Equations: A Study In *Three Nations*. In *Building connections: Theory, research and practice: Proceedings of the 28th annual conference of the Mathematics Education Research Group of Australasia*. Sydney: MERGA.
- WIJAYA, T.T., YING, Z., CHOTIMAH, S., BERNARD, M., 2020. Hawgent Dynamic Mathematic Software as Mathematics Learning Media for Teaching Quadratic Functions. In *Journal of Physics: Conference Series* (Vol. 1592, No. 1, p. 012079). IOP Publishing.

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