

## INTERDISCIPLINARY CONNECTION BETWEEN MATHEMATICS AND PHYSICS IN A GRAMMAR SCHOOL FOR GIFTED MATHEMATICS STUDENTS

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**Abstract.** Interdisciplinary connection of teaching content, as an approach that enables the deepening of knowledge in different subjects while simultaneously fostering various student competencies, is not sufficiently present in schools in the Republic of Serbia. Moreover, it is almost neglected in working with students gifted in mathematics. In this paper, we describe a two-lesson session on the topic Applications of Differential Calculus in Physics, in which we connected teaching content from Mathematical Analysis with Algebra and Physics in working with fourth-year high school students in Kragujevac, specifically those enrolled in the specialized mathematics program. Students had the opportunity to revisit and deepen previously acquired knowledge in Physics, and simultaneously recognize the application of differential calculus, which they had recently learned. The paper presents the problems solved by the students during the session, as well as the results of a survey, in which students evaluated this approach as useful, engaging, and they expressed that it increased their interest in studying mathematics and physics in greater depth.

*Keywords:* interdisciplinary connection; calculus; mathematics; physics; students gifted in mathematics

### 1. Introduction

The interdisciplinary connection between mathematics and physics is a profound testament to the synergy between these two sciences. While this relationship has a historical context, it can also be seen as a dynamic and continuous development that shapes our fundamental understanding of the world around us. On one hand, mathematics provides the language and tools

necessary for physicists to formulate, solve, and interpret various laws. At the same time, physics offers a rich source of inspiration and application for different mathematical theories (Boaler 2015).

The connection between mathematics and physics dates to ancient times, with Archimedes and Euclid making significant contributions to both disciplines. Later, during the scientific revolution, particularly through the works of Isaac Newton and other scientists of that era, this relationship became a cornerstone of modern science. Subsequently, the development of quantum mechanics and the theory of relativity further emphasized the deep interdependence of mathematics and physics. For example, Einstein's general theory of relativity heavily relies on the mathematical framework of differential geometry.

However, in education—at least in the Republic of Serbia – this connection seems to be underdeveloped and neglected in teaching and learning. When it comes to heterogeneous classrooms, teachers might justify this by arguing that certain students lack the necessary knowledge and skills to connect concepts from these two subjects. However, even when working with students enrolled in classes for those gifted in mathematics, this approach is not sufficiently present. Interdisciplinary integration of teaching content is recognized as an approach that fosters logical and critical thinking, collaborative skills, and creativity among high school students. With this paper, we aim to contribute to the interdisciplinary connection between mathematics and physics, specifically through the application of differential calculus in physics with high school students gifted in mathematics.

## **2. Theoretical background**

### ***2.1. Interdisciplinary Connection between Mathematics and Physics***

By analyzing research topics in the field of mathematics education, it is noticeable that insufficient attention has been given to studies exploring the connection between mathematics and other individual natural sciences (Michelsen 2005). When it comes to interdisciplinary connections with mathematics, physics emerges as one of the most natural choices due to the strong compatibility between these two subjects. However, certain authors

emphasize that this compatibility has not been adequately reflected in curricula (Park, Kim & Kang 2021). As previously mentioned, throughout history, different theories have pointed to the same conclusion: "the formulation of physics is mathematics" (Pospiech 2015). Mathematics can be perceived as the language through which physical theories are constructed, using symbols, relations, and operations to understand equations and their representations in physics (Buick 2007).

Interdisciplinary learning integrates various concepts and methodologies from multiple disciplines to provide a more comprehensive understanding of complex phenomena (Klein 1990). In the context of mathematics and physics, this approach is crucial due to the significant connection between these two subjects in educational content. Research indicates that students learning in interdisciplinary environments demonstrate a deeper understanding of concepts and retain them more effectively and for a longer period (Jacobs 1989). Additionally, studies have shown that integrating mathematical problem-solving techniques with physics concepts enhances students' conceptual understanding and improves their performance in both mathematics and physics (Redish & Steinberg 1999). Empirical research results suggest that interdisciplinary approaches enhance students' problem-solving skills by encouraging them to apply mathematical tools for analyzing and solving problems in a physics context (Hestenes 2010). In this way, students not only develop essential mathematical skills to a greater extent but also deepen their knowledge of physics. Other studies indicate that this approach also increases students' motivation for learning and their engagement in lessons (Boaler 2015). Hestenes (2010) researched the impact of instruction based on the mathematical modelling of physical concepts, and the findings revealed that students who participated in this integrated instruction achieved better academic results than their peers (Hestenes 2010).

### *2.2. Students gifted in mathematics*

A certain number of children have abilities that enable them to learn faster and more easily and to understand content more deeply than their peers (Borovik & Gardiner 2007; Ucar, Ucar & Cahskan 2017). Identifying individuals with such abilities and gaining a better understanding of how

they function is important for determining support measures to further develop their talents. According to Kenderov, the focus of the educational system on the average student results in mathematical talent and giftedness remaining underdeveloped or even undiscovered (Kenderov 2006). Some authors (Altaras Dimitrijević & Tatić Janevski 2016) believe that students with special abilities should be provided with appropriate environmental influences and that a specialized mathematics curriculum should be designed to offer tasks and activities that contribute to the development of gifted students' potential.

The grouping of students with high achievements in mathematics into special classes, and even special schools, emerged in the United States and some European countries nearly 100 years ago, with the primary goal of meeting the specific educational needs of these students (Mihajlović 2023). Grouping students with special abilities creates an environment in which they are stimulated by their peers of the same ability level, allowing them to process complex content more quickly and effectively (Neihart 2007; Swiatek & Lupkowski-Shoplik 2003). In a previously conducted meta-analysis (Kulik & Kulik 1987), out of 25 studies examining the effects of teaching gifted students grouped in specialized classes, 19 studies found that students in specialized classes achieved better results compared to mathematically gifted students who were in traditional, heterogeneous classrooms.

In the Republic of Serbia, the Law on the Fundamentals of the Education System recognizes the separation of mathematically gifted students into specialized classes. In seven high schools (grammar schools) across seven cities in Serbia, there are special classes for students with exceptional mathematical abilities. Students interested in attending these classes must take a qualifying exam. In addition to the entrance exam results, their academic performance during elementary school and achievements in mathematics competitions are also considered. Mathematics is studied much more extensively and intensively in these students' classes. Throughout all four years of high school, students have four weekly lessons in Mathematical Analysis with Algebra. In the first two years, they take Geometry with four weekly lessons. In the third year, they study Linear Algebra with Analytical

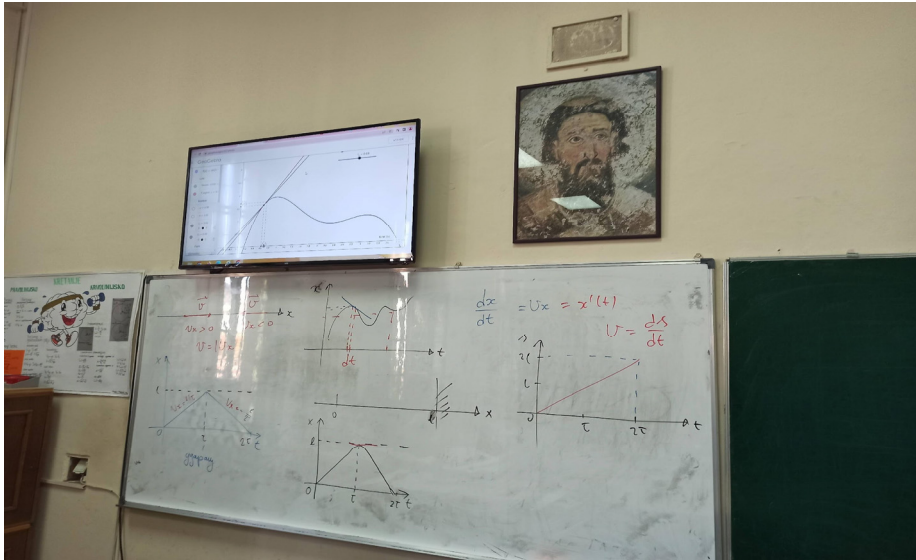
Geometry, while in the fourth year, they take Numerical Mathematics and Probability and Statistics, each with two weekly lessons. Thus, in each of the four years, students with special mathematical abilities have a total of eight weekly lessons in mathematical subjects. Additionally, it is important to highlight that the curriculums for Physics, Informatics and Programming subjects are also quite demanding for classes of students gifted in mathematics.

### **3. Present study**

Given that exemplary lessons for interdisciplinary connection of teaching content are rarely conducted in schools, and that they are almost never implemented in classes with students gifted in mathematics, we came up with the idea to hold an exemplary double lesson on the topic Applications of Differential Calculus in Physics. This double lesson was held on the last week of October 2023, in a class of fourth-year students gifted in mathematics at First Grammar School in Kragujevac. The lesson was attended by 14 students (the entire class) and 15 teachers (of mathematics, physics, informatics, and chemistry), as well as a pedagogist and psychologist from First Grammar School.

In the introductory part of the lesson, students reviewed the concept of derivatives, their geometric and mechanical interpretation, properties of derivatives, the derivative of a composite function, as well as Rolle's theorem and Lagrange's theorem. We used GeoGebra applets to illustrate some of these concepts during the introductory part of the classes. These GeoGebra applets were designed to allow students to visualize the definition of the derivative as the limit of the difference quotient – representing the ratio of the change in the function's value to the change in the input variable – as the increment approaches zero, thereby illustrating the concept of the tangent line to a function at a given point. The second applet enabled students to visually explore Rolle's and Lagrange's Mean Value Theorems. By selecting various functions and intervals over which the functions are continuous and differentiable, students could observe the existence of a point where the tangent line is parallel to the corresponding secant line (as stated

in Lagrange's Theorem), or, in the case of Rolle's Theorem, a point where the tangent line is horizontal, i.e., parallel to the x-axis.



**Figure 1.** Review of Derivatives and the Most Important Theorems of Differential Calculus

In the main part of the lesson, students solved specific problems illustrating the application of differential calculus to solving physics problems.

In the fourth-year class of students gifted in mathematics, nine students had an average grade of 5 (which is the highest grade) in the group of mathematics subjects in the fourth year (Mathematical Analysis with Algebra, Numerical Mathematics, and Probability and Statistics), three students had an average grade of 4, and two students had an average grade of 3. A similar distribution of grades was observed in physics: nine students had an average grade of 5, two students had an average grade of 4, and three students had an average grade of 3. No student had an average grade of 1 or 2 in either the mathematics group of subjects or physics.

Considering the small differences in student achievement in mathematics and physics, an individualized approach to teaching was applied. Each student worked on three problems aligned with their level of achievement. A total of seven problems (Irodov 2000) were prepared (three from mechanics, three from thermodynamics, and one from electromagnetism) for the class.

Below, we present three problems with solutions from the field of mechanics that students solved during the exemplary classes.

1. Two cars are moving at constant velocities with magnitudes  $v_1$  and  $v_2$  along two mutually perpendicular streets toward an intersection—point  $O$ , where the streets cross. At time  $t = 0$ , the cars were at distances  $l_1$  and  $l_2$  from point  $O$ . Determine after what time  $\tau$  from the initial moment the distance between the cars will be minimal and find the value of this minimum distance  $l_{min}$ .

*Solution:* If  $l_1$  and  $l_2$  are the initial distances of the first and second car from the intersection, and the magnitudes of their velocities are  $v_1$  and  $v_2$ , then after a time  $t$ , these distances are given by:

$$l_1(t) = l_1 - v_1 t, \quad l_2(t) = l_2 - v_2 t.$$

Using the Pythagorean theorem, the distance between the cars at time  $t$  is:

$$l(t) = \sqrt{(l_1 - v_1 t)^2 + (l_2 - v_2 t)^2}$$

Let  $\tau$  be the moment when the distance is minimized. The conditions for minimality are:

$$\left. \frac{dl(t)}{dt} \right|_{t=\tau} = 0 \quad \text{and} \quad \left. \frac{d^2l(t)}{dt^2} \right|_{t=\tau} > 0.$$

Applying differentiation rules and knowing that  $l_1$ ,  $l_2$ ,  $v_1$  and  $v_2$  are constants, we obtain:

$$0 = \frac{d\left(\sqrt{(l_1 - v_1\tau)^2 + (l_2 - v_2\tau)^2}\right)}{dt} \Rightarrow v_1(l_1 - v_1\tau) = -v_2(l_2 - v_2\tau).$$

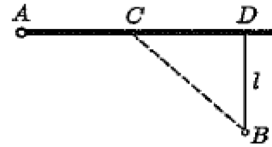
Solving for  $\tau$ , we get  $\tau = \frac{l_1v_1 + l_2v_2}{v_1^2 + v_2^2}$ . Thus, the minimum distance is:

$$l_{min} = l(t = \tau) = \frac{|l_1v_2 - l_2v_1|}{\sqrt{v_1^2 + v_2^2}}.$$

Notice that  $l_{min} = 0$  (which corresponds to a collision) if  $l_1v_2 = l_2v_1$ .

*Homework assignment:* Verify that the condition  $\frac{d^2l(t)}{dt^2}|_{t=\tau} > 0$  is satisfied.

2. From the point  $A$ , which is located on a road, it is necessary to reach the point  $B$ , which is situated on a flat meadow beside the road, in the shortest possible time by car. The car moves at a constant speed both on the road and across the meadow. Therefore, it must leave the road at the point (fig. 2).



**Figure 2.** Problem 2

If  $l$  is the shortest distance between points  $B$  and  $D$ , and if the speed of the car is  $n$  times greater when traveling on the road than when traveling across the meadow, determine the distance  $x$  between points  $C$  and  $D$  such that the travel time is minimized.

*Solution:* Let  $x$  denote the distance  $CD$ ,  $s$  the distance  $AD$ ,  $l$  the distance  $DB$ , and let  $v_2 = v$  be the speed of the car on the meadow. Then, the speed of the car on the road is  $v_1 = nv$  ( $n > 1$ ), while the distances traveled on the road and the meadow are  $s_1 = AC = s - x$  and  $s_2 = CB = \sqrt{l^2 + x^2}$ .

From this, the total travel time of the car is given by:

$$t(x) = \frac{s - x}{nv} + \frac{\sqrt{l^2 + x^2}}{v}.$$

The conditions for minimizing this time are:  $\frac{dt(x)}{dx} = 0$  and  $\frac{d^2t(x)}{dx^2} > 0$ .

Since  $s$ ,  $l$  and  $v$  are constant values, applying differentiation rules yields:

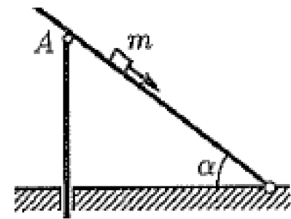
$$0 = -\frac{1}{nv} + \frac{x}{v\sqrt{l^2 + x^2}}$$

from which the required distance is obtained as:

$$x = \frac{l}{\sqrt{n^2 - 1}}$$

*Homework assignment:* Verify the condition  $\frac{d^2t(x)}{dx^2} > 0$  for  $x = \frac{l}{\sqrt{n^2 - 1}}$  and determine the minimum travel time  $t_{min} = t(x = \frac{l}{\sqrt{n^2 - 1}})$ .

**3.** A child slides down an inclined plane representing the slide. The vertical support and the endpoint of the slide are fixed, while point A can move vertically along the support depending on the value of angle  $\alpha$ . The coefficient of friction between the child and the slide is  $\mu = 0.14$ . For which value of the incline angle will the child reach the bottom in the shortest time (see fig. 3)?



**Figure 3.** Problem 3

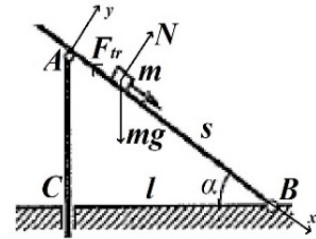
*Solution:* Let us orient the  $x$ -axis along the inclined plane and the  $y$ -axis perpendicular to it, directed upward. The equations of motion for the child along these axes are:

- $x$ -axis:  $ma = mg \sin \alpha - F_{tr}$
- $y$ -axis:  $N = mg \cos \alpha$ ,

where  $m$  is the mass of the child,  $\alpha$  is the incline angle of the plane, and  $mg$ ,  $F_{tr} = \mu N$ , and  $N$  are the magnitudes of the gravitational force, the kinetic friction force, and the normal force, respectively (these are the forces acting on the child in this system;  $g$  is the magnitude of gravitational acceleration).

From these equations, we obtain the acceleration of the child as:

$$a = g(\sin \alpha - \mu \cos \alpha).$$



**Figure 4.** Solution to Problem 3

We observe that the condition for the child to slide down the slide is  $a > 0$ , which means that for a given friction coefficient  $\mu$ , the condition  $\tan \alpha > \mu$  must be satisfied for any value of the acute angle.

Let the point  $C$  be on the horizontal plane, directly below the starting point  $A$ , and let  $l$  be the distance between this point and the base of the inclined plane  $B$ , while  $s$  is the distance traveled by the child from the starting point to the base. Note that according to the problem conditions,  $l$  is constant, whereas  $s = \frac{l}{\cos \alpha}$  is not constant, since the incline angle  $\alpha$  can change, while the point  $C$  always remains directly below the point  $A$  (fig. 4).

Since the child moves with uniform acceleration from rest, we have  $s = \frac{at^2}{2}$ . Solving for the sliding time, we obtain:

$$t(\alpha) = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2l}{g} (\sin \alpha \cdot \cos \alpha - \mu \cos^2 \alpha)^{-\frac{1}{2}}} = \sqrt{\frac{2l}{g} \left( \frac{1}{2} \sin 2\alpha - \mu \cos^2 \alpha \right)^{-\frac{1}{2}}}$$

The conditions for minimizing the child's sliding time are:  $\frac{dt(\alpha)}{d\alpha} = 0$  and  $\frac{d^2t(\alpha)}{d\alpha^2} > 0$ . Applying differentiation rules, we obtain:

$$0 = -\frac{1}{2} \sqrt{\frac{2l}{g}} \left( \frac{1}{2} \sin 2\alpha - \mu \cos^2 \alpha \right)^{-\frac{3}{2}} (\cos 2\alpha + 2\mu \sin \alpha \cos \alpha) \Rightarrow \cos 2\alpha + \mu \sin 2\alpha = 0$$

From this, we derive the required angle  $\tan 2\alpha = -\frac{1}{\mu} = -\frac{100}{14}$ .

Now we have  $2\alpha = -82^\circ$ . According to the conditions of the problem, the angle should lie in the first quadrant. Taking into account the periodicity of the function  $y = \tan 2\alpha$  and the physical formulation of the problem, the solution is  $\alpha = 49^\circ$ .

Below, we provide the formulations of the remaining 4 tasks.

4. We know that pressure  $p$ , temperature  $T$ , and air density  $\rho$  change with altitude. If we assume that at low altitudes above the Earth's surface,

pressure and air density are related by the equation  $\frac{p}{\rho^n} = \text{const}$ , which  $n = \text{const}$ , determine the temperature gradient  $\frac{dT}{dh}$ , i.e., the change in air temperature with altitude. The molar mass of air is  $M$ , the universal gas constant is  $R$ , and the gravitational acceleration is  $g = \text{const}$ . Consider air as an ideal gas.

5. Two moles of an ideal gas transition from state 1 to state 2 through a process that can be represented on the  $pV$  diagram by the line  $\frac{p}{a} + \frac{V}{b} = 1$ , where  $a = 1 \text{ MPa}$  and  $b = 16 \text{ l}$ . What is the maximum temperature  $T_{\text{max}}$  that the gas can reach in this process? The universal gas constant is  $R = 8.3 \frac{\text{J}}{\text{mol K}}$ .

6. A stationary container contains a piston connected to the right wall of the container by a light elastic spring. When the container is empty, the piston is pressed against the left wall of the container, and in this position, the spring (fig. 5) is undeformed. Then,  $n_m$  moles



Figure 5. Problem 6

of the container, where the molar isochoric heat capacity is  $C_V$ , while the right side remains in a vacuum. The gas is then slowly heated by a heater inserted through the right wall. If the container walls and the piston do not conduct heat, and friction is negligible, prove that the molar heat capacity  $C$  of the gas during this heating process is constant.

7. A thin ring of radius  $R$  is given, which is uniformly charged with a charge  $q > 0$ . Determine the dependence of the electric field intensity  $E$  on the axis of the ring, normal to its plane, at a distance  $l$  from its center. At what distance  $l_0$  from the center does the electric field intensity reach its maximum value  $E_{\text{max}}$ , and what is its value? Try to sketch the graph of the dependence  $E(l)$ .

#### 4. Students' impressions of the classes

In the final part of the lesson, a survey was conducted with the students to obtain their feedback. For this purpose, a questionnaire was created using Google Forms and distributed to the students during the lesson. The students were presented with statements related to the lesson itself and the interdisciplinary connection of teaching content. They were expected to respond honestly about the extent to which they agreed with the statements using a five-point Likert scale. Table 1 presents the students' responses.

**Table 1.** Results of the survey

Statement	I strongly disagree (n, %)	I somewhat disagree (n, %)	I am undecided (n, %)	I somewhat agree (n, %)	I strongly agree (n, %)	Total (n, %)
The lesson was interesting to me.	0 (0%)	1 (7.1%)	0 (0%)	5 (35.7%)	8 (57.2%)	14 (100%)
I believe that the lesson was useful.	0 (0%)	0 (0%)	0 (0%)	5 (35.7%)	9 (64.3%)	14 (100%)
I believe that the tasks I solved were difficult.	0 (0%)	7 (50%)	6 (42.9%)	1 (7.1%)	0 (0%)	14 (100%)
The work in the class increased my interest in physics.	0 (0%)	2 (14.3%)	2 (14.3%)	9 (64.3%)	1 (7.1%)	14 (100%)
The work in the class increased my interest in mathematics.	0 (0%)	1 (7.1%)	3 (21.4%)	9 (64.3%)	1 (7.1%)	14 (100%)
I would like teachers to hold joint lessons more often.	0 (%)	0 (%)	3 (21.4%)	7 (50%)	4 (28.6%)	14 (100%)

Based on the analysis of student responses, it can be observed that 13 out of 14 students found the lesson, in which they had the opportunity to see the application of mathematical concepts – specifically differential calculus in physics – interesting. All students consider the activities conducted during the lesson useful. Moreover, as many as 9 students completely agree with this statement. However, it cannot be said that the tasks students solved during the lesson were quite difficult for students. Specifically, 7 students disagreed with this statement, while 6 were undecided on the matter, and one student found the exercises difficult. The meaningfulness of integrating mathematical content – particularly Mathematical Analysis with Algebra – with physics is evident in the fact that 9 students somewhat agree that their interest in learning both mathematics and physics has increased as a result of the lesson. The other responses show a similar distribution. Students recognize the importance and benefits of interdisciplinary connections, as 11 students agree or completely agree that they would like teachers to collaborate more frequently and conduct joint lessons integrating different subjects. Three students were undecided on this question, while there were no negative responses on this topic.

In the last open-ended question, students were asked to answer which two subjects they would like their teachers to integrate into the lessons. The students' responses were as follows: Mathematical Analysis with Algebra and Physics (5 responses); Mathematical Analysis with Algebra and Informatics (1 response); Geometry and Physics (2 responses); Mathematical Analysis with Algebra and Probability and Statistics (1 response); Mathematical Analysis with Algebra and Numerical Mathematics (1 response); Physics and Biology (1 response). Three students did not provide an answer to this question.

## **5. Discussion and Conclusion**

By actively participating in the lesson, students gifted in mathematics solved physics problems from areas they had studied in previous years of their secondary education. Thus, the physics content was not new to them. However, what was novel in their approach was the application of differential calculus, which they had been introduced to in the lessons preceding the described exemplary lesson. Previously, students solved given problems by

estimating the values of algebraic expressions and using algebraic transformations such as the square of a binomial and the difference of squares to determine the maximum and minimum values of the required expressions to answer the given questions successfully.

After acquiring knowledge of differential calculus, students had the opportunity to see how it could be applied, particularly in determining the extreme values of functions of a single variable, to solve physics problems. This significantly simplified the problem-solving process, making the solutions more elegant. By applying mathematical analysis in solving physics problems, students had the chance to develop problem-solving competencies while simultaneously reinforcing their physics knowledge (Hestenes 2010).

Students had a positive opinion of the exemplary lesson, stating that their interest in learning both mathematics and physics had increased. This aligns with previous research findings, which suggest that interdisciplinary connections between mathematics and physics enhance students' interest in learning (Boaler 2015). Moreover, students expressed various ideas about which subjects teachers could integrate into interdisciplinary research, demonstrating their awareness that school subjects are not isolated but can be meaningfully connected.

With this paper, we aim to enrich the body of research indicating that integrating mathematics and physics, while focusing on the development of problem-solving skills, critical thinking, and transferable skills, can help educators foster a deeper understanding among students, increase their motivation, and enhance their engagement. The main advancement compared to previous studies is that most research has focused on students in heterogeneous classrooms, whereas we examined a homogeneous group—students gifted in mathematics. We believe that the problems presented in this paper will be useful to educators in planning lessons that are based on the interdisciplinary integration of mathematics and physics. A potential focus for future research could be the interdisciplinary connection between trigonometry and physics for younger high school students, with findings discussed in collaboration with educators, curriculum designers, and policymakers to enhance STEM engagement among high-achieving students.

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