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EXPLORING PICK'S THEOREM IN THE CLASSROOM: A COMPARISON BETWEEN TRADITIONAL AND DIGITAL METHODOLOGIES USING PICKSOLVER SOFTWARE

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Abstract. This paper presents a study that analyzes how traditional methods, based on the use of paper-and-pencil (PPB), can be integrated with digital tools to solve geometric problems using Pick's theorem. Students worked in groups of four and completed four tasks of increasing complexity, solving each one first with traditional methods and then using PickSolver, a custom software designed to compute polygonal areas and identify maximum-area polygons within lattice polygons, even those containing holes. Results highlight distinct advantages: while PPB promoted deeper conceptual understanding and more deliberate reasoning, PickSolver enabled quick verification and enhanced student motivation. Rather than being opposing strategies, both approaches worked best when combined. Their integration created a more engaging and visually rich learning environment. The results indicate that the integration of both methods supports more robust and meaningful mathematical learning.

Keywords: geometry education; lattice polygon; Pick's theorem; area; digital tools

1. Introduction

Mathematics education has undergone significant changes over the last century, especially with the introduction of technology in the classroom. This advancement has fostered innovative methodologies and enhanced access to educational resources. In particular, technological tools such as Dynamic Geometry Software (DGS) (Chan & Leung 2014; Greefrath et al. 2018; Hölzl 2001), including GeoGebra (Juandi et al. 2021; Yohannes & Chen 2023; Radovic et al. 2020), Cinderella (Richter-Gebert et al. 2012; Kurtenbach et al. 2014), Cabri (Bokosmaty et al. 2017; Yazlik & Ardahan 2012), and the Geometer's Sketchpad (Guven & Karatas 2009; Bosse 2018), along with interactive platforms such as Khan Academy (Vidergor & Ben-Amram 2020) and Brilliant (de la Puente & Perez 2023), are improving education by providing visual representations of mathematical concepts that were previously difficult to assimilate. One of the main reasons this transformation has taken place is undoubtedly the use of computers, which have become indispensable in today's classrooms, as they foster interaction and collaboration among students (Haleem et al. 2022).

Several studies have emphasized the importance of incorporating technology into mathematics education. For example, Weigand et al. (2024) argue that digital technology has gained increasing relevance in the field of mathematics education. Similarly, Engelbrecht & Borba (2024) suggest that these tools can enhance concept learning by enabling students to explore mathematics in a more concrete way. Furthermore, Hillmayr et al. (2020) point out that technology can be particularly beneficial for students who struggle with mathematics, as it provides additional resources that support practice in a less intimidating environment. In this context, the debate over whether technology should be used in mathematics instruction has been the subject of numerous studies and publications (Drijvers 2015; Myers 2009; Viberg et al. 2023). Accordingly, authors such as Leung & Chan (2007) caution that technology should not be considered a complete substitute for traditional teaching methods.

In the context of geometry education, dynamic geometry software (DGS) (Chan & Leung 2014; Greefrath et al. 2018; Hölzl 2001) has gained popularity. These tools provide the opportunity not only to simulate constructions, but also to discover geometric relationships and proofs in a visual and interactive format. They actively engage students in the learning process, which greatly contributes to the development of geometric thinking. Although there is no doubt that integrating technology into the learning process is beneficial, it must be used with great care, as improper implementation can undermine the relationship between teacher and student.

Building on this perspective, the present paper introduces PickSolver, geometry software specifically designed to solve area calculation problems in lattice polygons, based on Pick's theorem (Pick 1899; Varberg 1985; Papadopoulos & Iatridou 2010). PickSolver enables, first, the calculation of the area of any lattice polygon, including those with holes. Second, the software offers the possibility of determining the simple k-gon of the maximum area inscribed within a given polygon, where k represents the number of sides specified by the user.

One of the main innovations of PickSolver lies in its versatility: it does no require different methods to find, for example, the largest triangle, the max imum area quadrilateral, the maximum area pentagon, or any other simpl polygon. A single underlying algorithm is capable of efficiently computin all these solutions. The user simply specifies the desired number of sides an the type of optimization required: maximum area, or a simple and conve polygon.

The PickSolver software can be applied to multiple real-world applica tions, including solar panel installation (Awasthi et al. 2020), medical imag analysis (Ali et al. 2021; Gibert et al. 2013; Alqazzaz et al. 2019; Menz et al. 2014), robotics (Bast & Hert 2000; Wei et al. 2021), and geographi information systems (GIS) (Girard et al. 2021; Xu et al. 2022). The com plete software package is openly accessible via the Zenodo repository (Molan 2025).

The classroom methodology integrated the traditional paper-and-penci method (PPB) for solving geometry problems with the digital use of th PickSolver software. In this way, students were required to complete th activities using both methods and then compare the results obtained. Th first method allows students to identify and reason about geometric element without the support of digital tools, promoting a more direct understandin of the concepts. In contrast, the use of the digital tool enables them to verif the accuracy of the results and solve problems quickly and without errors As a result, students not only enhance their conceptual understanding of th problem but also begin to develop a critical perspective on both methods In doing so, they acquire technological skills (Serin 2023) that are equall important for their overall education.

Unlike conventional methods, which rely purely on manual procedures technology provides visual and interactive learning environments in whicl both teachers and students can experiment, represent, and reflect on abstrac ideas, progressing at their own pace (Kennewell et al. 2008; Wood & Ash field 2008). These tools not only increase the motivation and engagement o students, but also enhance the teaching and learning process as well as th development of personal strategies for solving complex problems.

This paper explores the educational potential of technological tools in th teaching of geometry through a practical case study: the PickSolver software Following a review of Pick's theorem, which provides the theoretical founda tion for the proposal, the functionality and main features of the software ar described. The methodology used to assess the learning outcomes of a grou of students is presented, comparing traditional and digital methods. The pa per includes a dedicated results section, where the most relevant findings o the study are analysed, followed by the conclusions.

2. Preliminary

2.1. Lattice polygon

We define a *lattice polygon* (Molano et al. 2022, 2023) as a simple polygon whose points have integer coordinates and which has the following properties:

1. It is defined on a *regular partition* $\Pi = \Pi_x \times \Pi_y$ of order $r \times s$ formed by $r+1$, $s+1$ equally spaced points, with $a, b, c, d \in \mathbb{Z}$, that satisfy:

$$\begin{aligned} \Pi_x &= \{a = x_0 < x_1 < \ldots < x_r = b\} \\ \Pi_y &= \{c = y_0 < y_1 < \ldots < y_s = d\} \end{aligned}$$

2. The connections between consecutive vertices are not necessarily established in the eight directions, $\pi k/4, k = 0, \ldots, 7$.

We denote by $G_L = \{(x_i, y_j) : 0 \leq i \leq r, 0 \leq j \leq s\}$, the square grid composed of points of the partition Π. We define *partition size*, $L = |x_{i+1} - x_i| = |y_{j+1} - y_j|$, the length of the side of each square formed by the square grid.

Fig. 1 illustrates a lattice polygon P with two holes, H_1 and H_2, constructed on a regular partition of order 7×8 with partition size L.

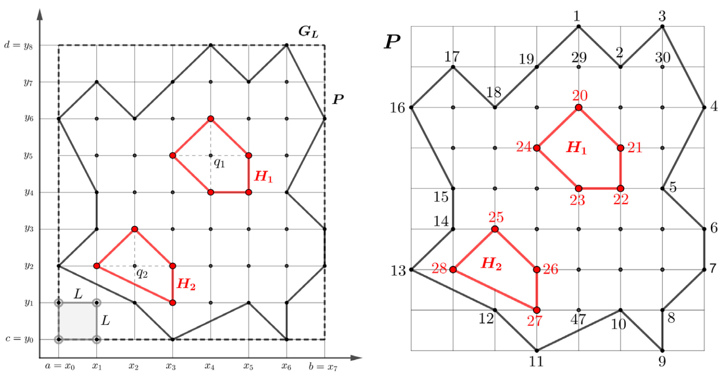


Figure 1. Construction of the lattice polygon

2.2. Pick's theorem

Pick's theorem, formulated by Georg Alexander Pick in 1899 (Pick 1899), provides a simple formula for computing the area of a lattice polygon P, possibly with holes:

$$A(P) = \left(\#(\imath P) + \frac{\#(\partial P)}{2} - 1 \right) \cdot L^2$$

where $A(P)$ denotes the area of lattice polygon P, # represents the cardinality of a set, ∂P is the set of boundary nodes of P, and its complementary in P, $\imath P$, the interior points, so that $P = \partial P \cup \imath P$.

If P has h holes (Varberg 1985), the formula generalizes to:

$$A(P) \;=\; \left(I + \frac{B}{2} + h - 1\right) \cdot L^2 \tag{1}$$

where $I = I_0 - \sum_{i=1}^{h}(I_i + B_i)$ and $B = B_0 + \sum_{i=1}^{h} B_i$ are the number of interior ($\imath P$) and boundary (∂P) points of P, I_0 and B_0 the number of interior ($\imath P_0$) and boundary (∂P_0) points of P_0 (outer lattice polygon) and I_i and B_i the number of interior ($\imath H_i$) and boundary (∂H_i) points of each hole H_i, $1 \leq i \leq h$.

Thus, for fig. 1, we obtain:

$$A(P) \;=\; \left(19 + \frac{28}{2} + 2 - 1\right) \cdot L^2 = 34\, L^2,$$

since:

$$\begin{cases} \partial P &= \partial P_0 \cup \partial H_1 \cup \partial H_2 = \{p_1, \ldots, p_{19}, p_{20}, \ldots, p_{24}, p_{25}, \ldots, p_{28}\} \\ \imath P &= \imath P_0 - (H_1 \cup H_2) = \{p_{29}, \ldots, p_{47}\} \\ \partial H_1 &= \{p_{20}, p_{21}, p_{22}, p_{23}, p_{24}\} \\ \imath H_1 &= \{q_1\} \\ \partial H_2 &= \{p_{25}, p_{26}, p_{27}, p_{28}\} \\ \imath H_2 &= \{q_2\} \end{cases}$$

An alternative way to compute the area would have been to use the expression:

$$A(P) \;=\; A(P_0) - \sum_{i=1}^{h} A(H_i) \tag{2}$$

where the area of each polygon and each hole is calculated separately. Thus:

$$A(P) \;=\; 38,5L^2 - 2,5L^2 - 2L^2 = 34\, L^2$$

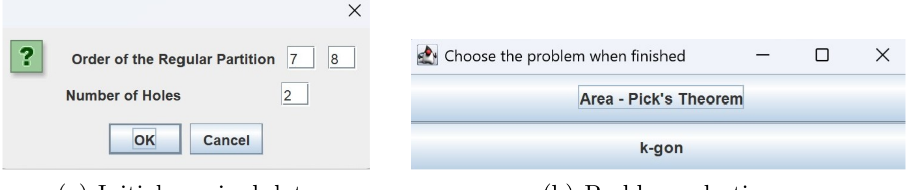
3. PickSolver

PickSolver was developed to meet both an educational and computational need: to facilitate the calculation of areas in lattice polygons, including those with holes, and to find the simple or convex k-gon of maximum area that can be inscribed in a given lattice polygon. While most Dynamic Geometry Software (DGS) programs (Juandi et al. 2021; Yohannes & Chen 2023; Radovic et al. 2020; Richter-Gebert et al. 2012; Kurtenbach et al. 2014; Bokosmaty et al. 2017; Yazlik & Ardahan 2012; Guven & Karatas 2009; Bosse 2018) offer a wide range of features for general geometric constructions, none have been specifically designed to optimize areas in lattice polygons based on Pick's theorem. This purpose justifies the dual objective pursued by PickSolver: automating calculations that would otherwise be too laborious, and developing an educational tool that enables students to learn through visualization and direct interaction with geometric objects.

PickSolver features an intuitive design that significantly reduces the learning curve, making it accessible for both students and teachers. Users can focus on analyzing and understanding geometric concepts instead of being burdened by purely technical aspects.

3.1. Construction of the lattice polygon

When the *PickSolver* application is executed, an initial window appears (fig. 2a), designed to construct the lattice polygon over a regular partition. The user must fill in the required fields: the order of the regular partition and the number of holes. Additionally, the user can choose either to calculate the area of the lattice polygon or to determine the k-gon with maximum area inscribed within it (fig. 2b).



(a) Initial required data (b) Problem selection

Figure 2. Construction of the lattice polygon

To construct the lattice polygon, the user selects an initial point on the partition and then sequentially selects all the points that form the boundary of the desired polygon (fig. 3). The construction is completed when the first selected point is clicked again, thus closing the contour. The same procedure is used for creating holes.

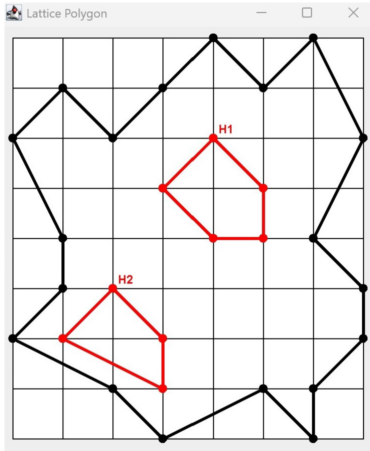


Figure 3. Lattice polygon

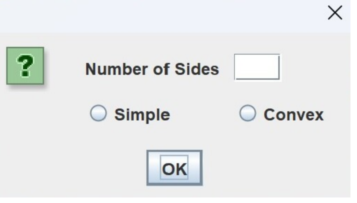
If the process is successful, the following sets are obtained, with each point defined by coordinates stored in $Points$:

$$\begin{cases} Polygon = \partial P_0 = \{1, 2, \ldots, 19\} \\ HolesP = \{H_1, H_2\} = \{\{20, 21, 22, 23, 24\}, \{25, 26, 27, 28\}\} \\ Points = P = \{1, \ldots, 47\} = \{(4,8), (5,7), (6,8), (7,6), (6,4), (7,3), (7,2), \\ \qquad (6,1), (6,0), (5,1), (3,0), (2,1), (0,2), (1,3), (1,4), (0,6), (1,7), \\ \qquad (2,6), (3,7), (4,6), (5,5), (5,4), (4,4), (3,5), (2,3), (3,2), (3,1), \\ \qquad (1,2), (4,7), (6,7), (1,6), (3,6), (5,6), (6,6), (1,5), (2,5), (6,5), \\ \qquad (2,4), (3,4), (3,3), (4,3), (5,3), (6,3), (4,2), (5,2), (6,2), (4,1)\} \\ L = 1 \\ N = 47 \end{cases}$$

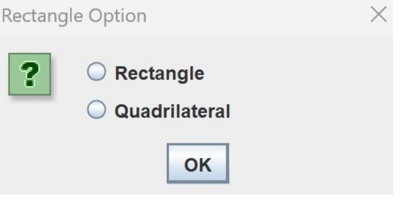
3.2. Solutions

Fig. 4a shows the available options for computing different solutions after selecting the *k-gon option* from fig. 2b. Specifically, the user can define the number of sides of the polygon and specify whether the resulting polygon should be simple or convex. Furthermore, if a four-sided polygon is selected, the user can choose to compute either a rectangle or a general quadrilateral.

Once the user has defined the desired k-gon along with the relevant options, they press the “OK” button to initiate the calculation and obtain the solutions.



(a) Initial options



(b) Rectangle/Quadrilateral

Figure 4. k-gon option

- `Save.` Save the solution image in the "Images" folder.
- `Next.` Show the next solution if there is more than one solution available.
- `Previous.` Show the previous solution if there is more than one solution available.
- `OK.` Finish the process and inquire whether another solution needs to be calculated.

Moreover, all computed solutions are stored in the `Solutions.txt` file which contains the coordinates of the polygons, the number of solutions, the area, and the computation time for each solution.

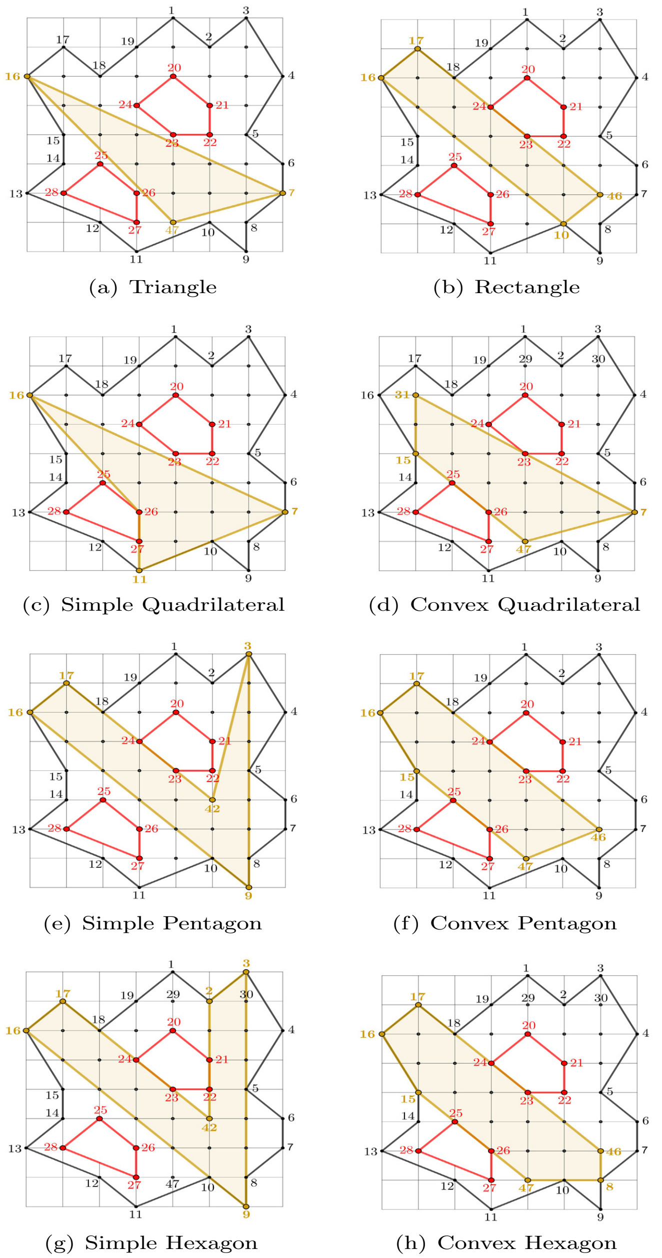
3.3. Examples of solutions computed with PickSolver

Table 1 presents the different solutions obtained by applying the PickSolver software to the polygon shown in fig. 1. Additionally, the corresponding solutions are graphically represented in fig. 5.

4. Methodology

4.1. Participants and context

Twenty students aged between 13 and 14 participated in the project voluntarily and with the informed consent of their legal guardians. The activity was carried out in a classroom equipped with personal computers, one per student, and an interactive digital whiteboard (IDW). These resources provided a technologically suitable environment for the implementation of the project (De Vita et al. 2014; Holmes 2009). Prior to the sessions, the PickSolver software was installed and its correct functioning verified, ensuring its availability on all devices.



(a) Triangle (b) Rectangle

(c) Simple Quadrilateral (d) Convex Quadrilateral

(e) Simple Pentagon (f) Convex Pentagon

(g) Simple Hexagon (h) Convex Hexagon

Figure 5. Solutions: Maximum-area simple and convex k-gon

Table 1. Solutions: Maximum-area simple and convex k-gon

k-gon	Solution	Area
Triangle	(7,47,16)	9.5
Rectangle	(10,46,17,16)	10
Simple Quadrilateral	(7,16,26,11), (7,31,26,11), (7,47,15,31), (15,47,46,17), (16,47,46,17)	12
Convex Quadrilateral	(7,47,15,31), (15,47,46,17), (16,47,46,17)	12
Simple Pentagon	(3,42,17,16,9), (7,31,14,40,11), (7,31,15,26,11), (7,31,16,26,11), (7,31,28,39,11), (11,46,17,16,26)	14
Convex Pentagon	(15,47,46,17,16)	13.5
Simple Hexagon	(2,42,17,16,9,3), (3,42,17,15,47,8), (3,42,17,16,47,8), (7,23,17,16,26,11), (7,47,15,17,42,33), (7,47,16,17,42,33)	16
Convex Hexagon	(8,47,15,16,17,46)	14.5

Previously, the students had worked in the classroom on topics such as polygons, the area of basic geometric figures, triangles, quadrilaterals, regular polygons, Pick's theorem, and the representation of points in the Cartesian plane. This foundational knowledge was essential for carrying out the proposed activities.

The students were organized into four groups of five, enabling collaborative learning and the exchange of ideas. A total of four 55-minute sessions were held over the course of two weeks. In each session, the groups completed a specific task of increasing difficulty, using both traditional paper-and-pencil (PPB) methods and the PickSolver software. This combination allowed for a direct comparison between the two methods and enabled drawing conclusions regarding their effectiveness.

The traditional method consisted mainly of manually representing polygons, calculating the area using Pick's theorem, or, given a value k specified in the problem statement, determining the simple or convex k-gon with the largest area. Since the difficulty of the problems increased with the number of sides of the polygon to be constructed, collaborative learning played a key role in enabling students to successfully tackle and solve each of the proposed tasks. Numerous studies support the idea of Collaborative Learning (CL) as

an effective tool in the teaching–learning process (Laal & Ghodsi 2012; Swa
2006; Moss & Beatty 2006; Smith & MacGregor 1992).

Subsequently, using the PickSolver software, the students recreated th same polygons within the program's interface, obtaining the solutions auto matically and visually. The Interactive Digital Whiteboard (IDW) was use consistently as a support tool to clarify doubts, encourage participation, an promote greater student engagement throughout the activity.

The final ten minutes of each session were allocated for each group t reflect critically on the effectiveness of the methods used, the conceptual un derstanding achieved, and their personal perceptions when working with bot the traditional and digital methods. They also reflected on the collaborativ work conducted to complete the various tasks.

At the end of each session, all worksheets produced by the students usin the traditional method were collected, along with the session recordings mad using Camtasia Studio (Demyan 2014). These recordings captured all studen interactions with the software, thereby facilitating the evaluation process car ried out by the teacher, including the analysis of strategies used, errors made and decisions taken during problem-solving.

4.2. Workshop planning and execution

In this subsection, we present the four tasks proposed to the student during the intervention, arranged in order of increasing difficulty. These task required students to apply Pick's theorem both manually, using the paper and-pencil method (PPB), and digitally, with the support of the PickSolve software. The sequence is described in Table 2.

4.2.1. Task 1: Lattice polygon

The first task aims to improve understanding of lattice polygons, Pick' theorem, and the PickSolver application.

Task 1a. Calculate the area of the lattice polygon shown in fig. 6a usin Pick's theorem, PPB method. Then, verify the result using the PickSolve software.

Task 1b. Determine the triangle of maximum area inscribed within th region shown in fig. 6b by PPB method (two solutions). Then, use PickSolve to compute the optimal solutions. Discuss the differences in time, accuracy and strategy between both methods.

4.2.2. Task 2: Lattice polygon with holes

This task extends the previous activity by incorporating lattice polygon with holes. It enables students to work with more elaborate geometric struc tures and observe how PickSolver automatically and efficiently handles inter nal voids.

Table 2. Sequence of instructional sessions and tasks

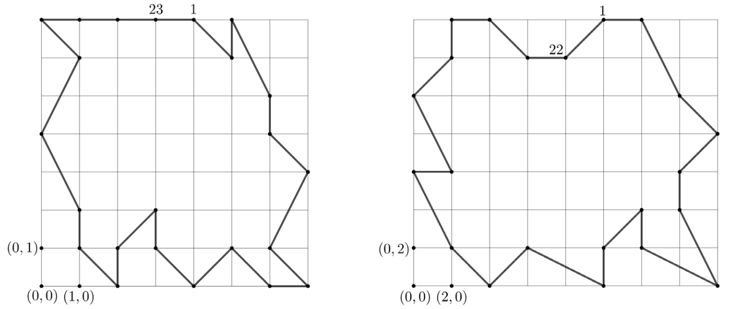
Session	Task
S_1	Presentation of the workshop (5 min) Task 1a (10 min): Area of the lattice polygon Task 1b (30 min): Triangle of maximum area Final Reflection (10 min)
S_2	Session presentation (5 min) Task 2a (30 min): Coordinates and area of the lattice polygon with holes Task 2b (10 min): Rectangle of maximum area Final Reflection (10 min)
S_3	Session presentation (5 min) Task 3a (20 min): Construct the lattice polygon P, including its three holes Task 3b (20 min): Area of the lattice polygon Final Reflection (10 min)
S_4	Session presentation (5 min) Task 4a (10 min): Coordinates of the lattice polygon Task 4b (30 min): Rectangle, triangle, and simple pentagon of maximum area Final Reflection (10 min)

Task 2a. Determine the coordinates of the lattice polygon and its holes shown in fig. 7a, and compute the area using Pick's theorem. Solve it manually and then verify the result with PickSolver.

Task 2b. Compute the rectangle of maximum area inscribed in fig. 7b, using both manual methods and PickSolver (one solution).

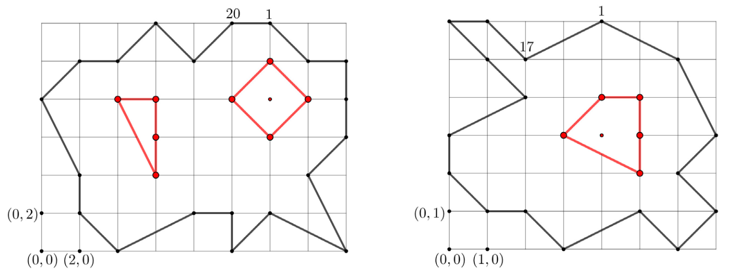
4.2.3. Task 3: Construction of lattice polygon with multiple holes

In this task, a set of points defines the outer boundary of a lattice polygon P and three internal holes. The objective is to construct the complete lattice polygon and analyze it using both manual methods and digital tools



(a) Area Pick's theorem (b) k-gon

Figure 6. Task 1. Lattice polygon



(a) Area Pick's theorem (b) k-gon

Figure 7. Task 2. Lattice polygon with holes

$$\begin{cases} Order\ 10 \times 10 \\ Polygon = \partial P_0 = \{1, \ldots, 31\} = \{(5,10),(7,9),(9,10),(10,7), \\ \quad (9,6),(8,5),(9,4),(9,3),(9,2),(10,0),(8,1),(7,0),(6,1), \\ \quad (5,1),(4,1),(3,0),(2,1),(0,0),(1,1),(1,2),(0,3),(2,4), \\ \quad (3,5),(2,7),(3,8),(2,8),(1,8),(0,10),(2,9),(3,10),(4,9)\} \\ HolesP = \{H_1, H_2, H_3\} \\ \quad H_1 = \{32,33,34,35\} = \{(6,7),(7,8),(8,7),(7,6)\} \\ \quad H_2 = \{36,37,38,39,40,41,42,43\} = \\ \qquad = \{(3,3),(4,4),(4,5),(5,4),(6,3),(6,2),(5,2),(4,3)\} \\ \quad H_3 = \{44,45,46,47\} = \{(7,3),(8,4),(8,3),(8,2)\} \\ L = 1 \end{cases}$$

Task 3a. Represent the given points on a regular partition 10×10 and construct the lattice polygon P, including its three holes.

Task 3b. Calculate the area of the lattice polygon P by applying Pick's theorem using both the traditional method and the PickSolver software.

4.2.4. Task 4: Solar panels

Solar panels are an efficient and environmentally friendly source of electrical energy. Fig. 8a shows an image of a plot of land partially covered with solar panels, highlighting a new region C intended to expand the photovoltaic installation by utilizing the largest possible surface area. Furthermore, fig. 8b illustrates the lattice polygon P, and fig. 8c defines its coordinates.

Task 4a. Calculate the coordinates of the lattice polygon P.

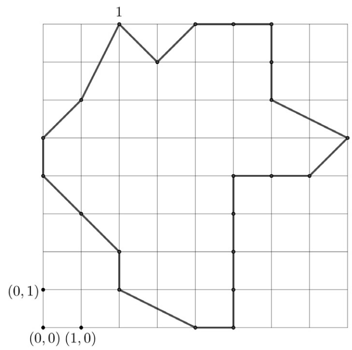
Task 4b. Determine the triangle, rectangle, and the simple pentagon of maximum area that can be inscribed in P (one solution). Solve the problem manually, and then verify the results using PickSolver.

5. Results

In this section, the results obtained by the four selected groups are presented. The results of Group 2 were selected for presentation, as the group stood out during the sessions due to its notable collaborative learning dynamics. Group 2 demonstrated effective internal organization, with each member taking on a clearly defined role: one was responsible for drawing the lattice polygon and representing the figures; another performed the calculations; a third used the PickSolver software; and the others validated the results and suggested improvements to optimize the problem-solving process. This organizational structure promoted collaborative learning among the group members, enabling them to successfully complete all the tasks assigned.

(a) Solar panels

(b) Closed contour C



(c) Lattice polygon P

Figure 8. Task 4. Solar panels

5.1. Task 1: Lattice polygon

The first task required the students to work with a lattice polygon. First, they manually calculated the area by applying Pick's theorem (Task 1a) (fig. 9), and then they determined the triangle with the largest area inscribed within it (Task 1b) (fig. 10). Cooperation among the members of Group 2 proved essential, as they had to count the interior and boundary points with great precision.

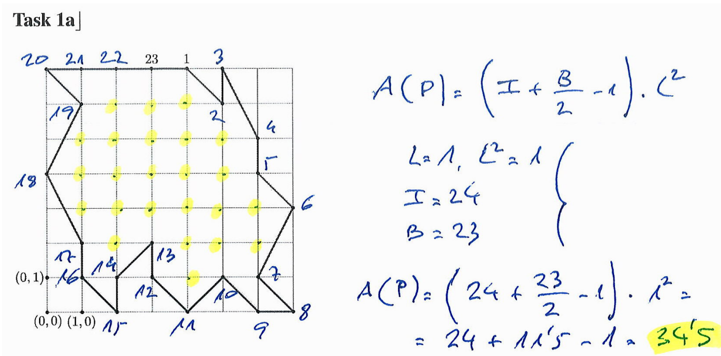


Figure 9. Task 1a. Area of the lattice polygon

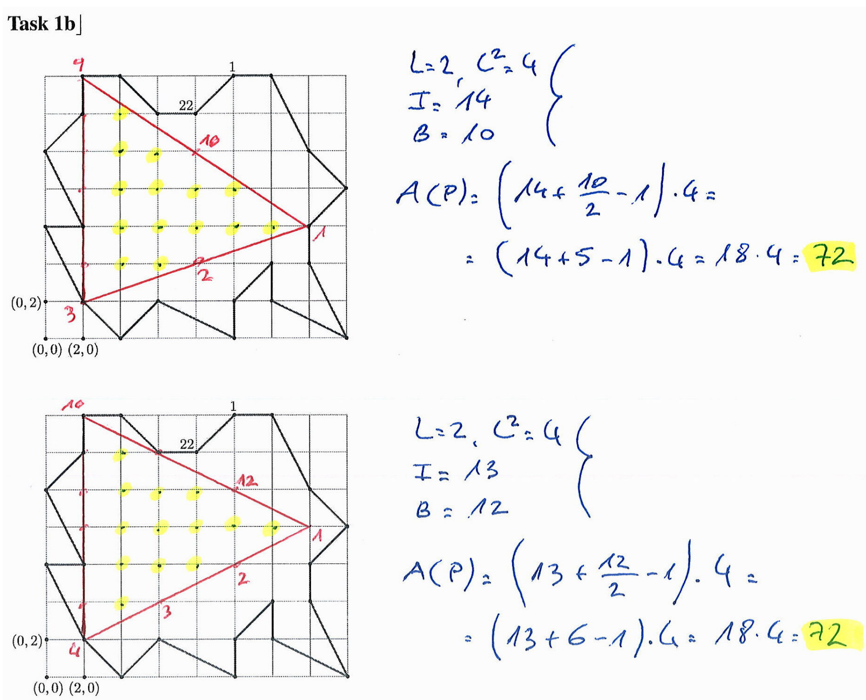


Figure 10. Task 1b. Triangle of maximum area (PPB method)

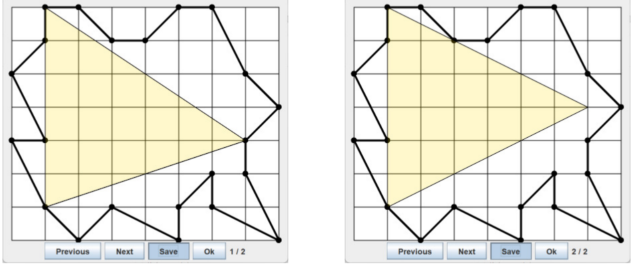


Figure 11. Task 1b. Triangle of maximum area (PickSolver)

The use of PickSolver in Task 1b (fig. 11) was beneficial for the group, as it allowed the students to confirm their results. Furthermore, it offered the opportunity to experiment with different configurations by calculating other types of polygons not required by the problem. Carmen, the representative of Group 2, commented on her experience: “It was easy to identify our initial mistakes and, at the same time, it motivated us to keep trying to solve them.”

5.2. Task 2: Lattice polygon with holes

Compared to the previous task, the second one was more complex due to the inclusion of holes in a lattice polygon. First, the students identified the coordinates of both the outer polygon and the holes, and then applied Pick’s theorem to calculate the total area (fig. 12). All members of Group 2 worked in a coordinated manner: while some students located the interior and boundary points, others carried out the necessary calculations to determine the area using the two formulas. This division of responsibilities was beneficial for the group, as it facilitated the collective verification of the results.

The main difficulty encountered by Group 2 was applying the generalized formula (Formula 1), due to the complexity of correctly incorporating the points. This difficulty was resolved by the group and subsequently verified using the PickSolver software.

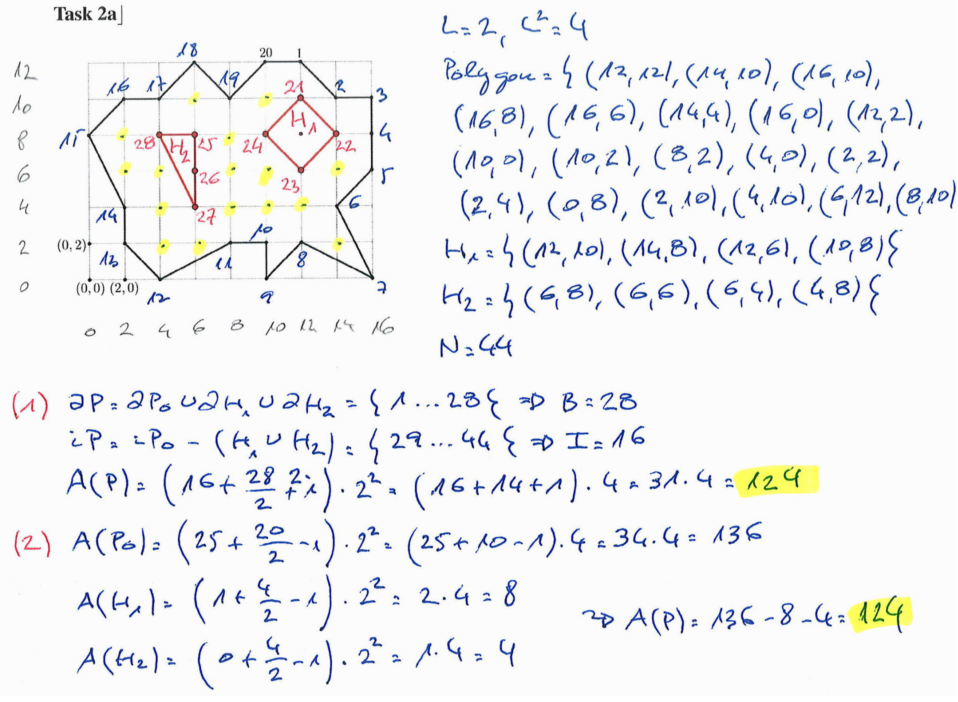
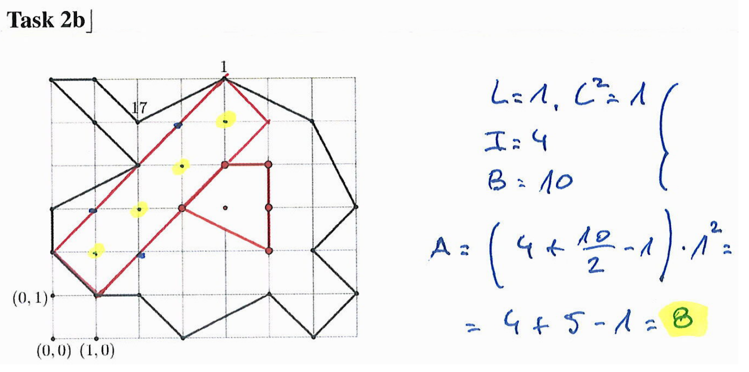
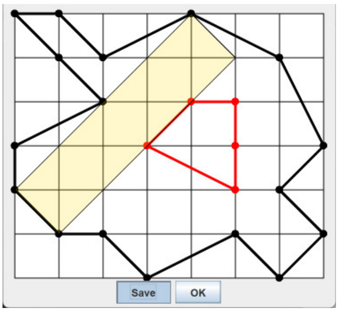


Figure 12. Task 2a. Coordinates and area of the lattice polygon with holes

During the second part of the task (Task 2b), the students calculated the largest area rectangle inscribed in a lattice polygon. The members of Group 2 explored various configurations, visually and analytically estimating their areas, and ultimately arrived at the correct result (fig. 13a). PickSolver proved to be very useful, as it allowed the automatic generation of the solution (fig. 13b), thereby validating the result obtained manually. Finally, the software enabled the exploration of alternative configurations that had not been considered during the initial phase of the task.



(a) PPB method



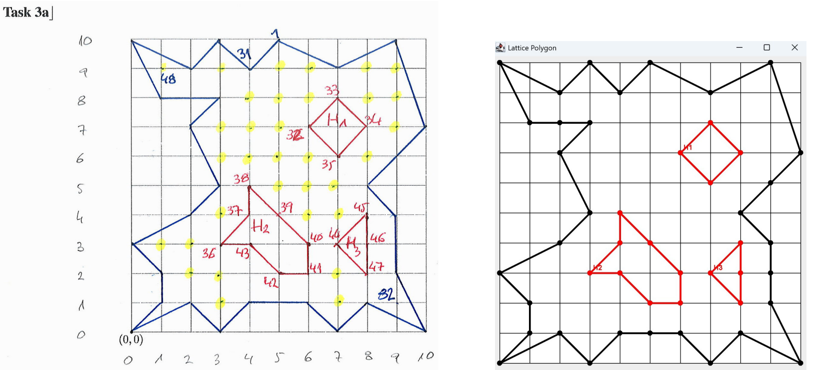
(b) PickSolver

Figure 13. Task 2b. Rectangle of maximum area

5.3. Task 3: Construction of lattice polygon with multiple holes

In the third task, students represented, on a regular partition of order 10×10, a lattice polygon that included three holes (Task 3a). Initially, this representation was carried out manually (fig. 14a), which demanded a high level of precision, as any error in the point coordinates could negatively affect the subsequent area calculation. Later, all the points were entered into the PickSolver software (fig. 14b), reaching the same conclusion. Group 2 performed outstandingly, as they organized themselves in a collaborative and efficient manner, which facilitated the accurate construction of the polygon and its elements.

In the second part (Task 3b), students used Pick's theorem to calculate the area of the previously constructed lattice polygon (fig. 15). Group 2 correctly applied both versions of the formula and subsequently used the PickSolver software to verify the result obtained manually.



(a) PPB method (b) PickSolver

Figure 14. Task 3a. Construct the lattice polygon P

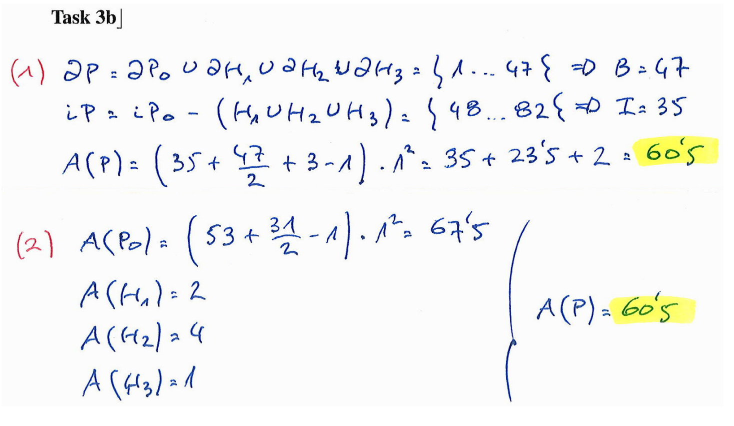


Figure 15. Task 3b. Area of the lattice polygon

5.4. Task 4: Solar panels

The final task was the most engaging for the students, as it presented a real-world scenario: calculating the optimal plot of land for the installation of solar panels. Based on an aerial image (fig. 8a) and the constructed lattice polygon (fig. 8c), students solved two geometric problems: determining the coordinates of the lattice polygon (Task 4a) and identifying the maximum area of different polygons that could be inscribed within it (Task 4b).

Group 2 correctly identified all the coordinates of the polygon (fig. 16) and also completed the second part of the task effectively (fig. 17). The use of PickSolver proved particularly decisive for the group due to the speed with which it generated the solutions (fig. 18), which also allowed them to compare the manually obtained results with those provided by the software.

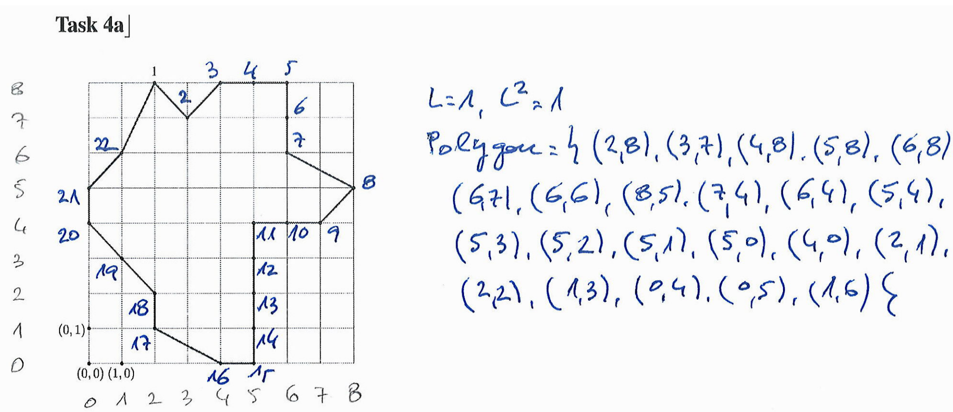


Figure 16. Task 4a. Coordinates of the lattice polygon

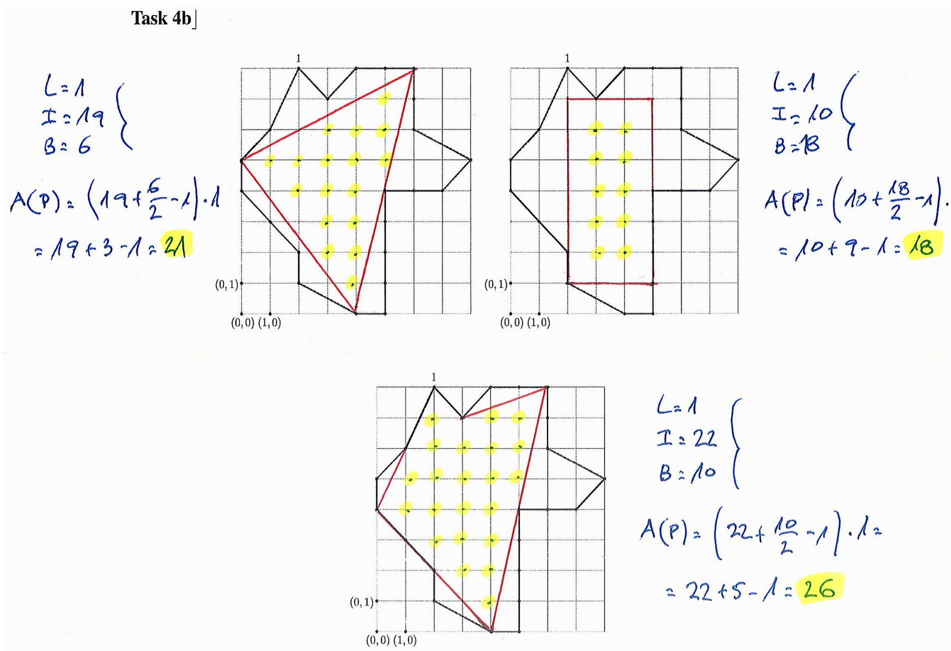


Figure 17. Task 4b. Rectangle, triangle, and simple pentagon of maximum area (PPB method)

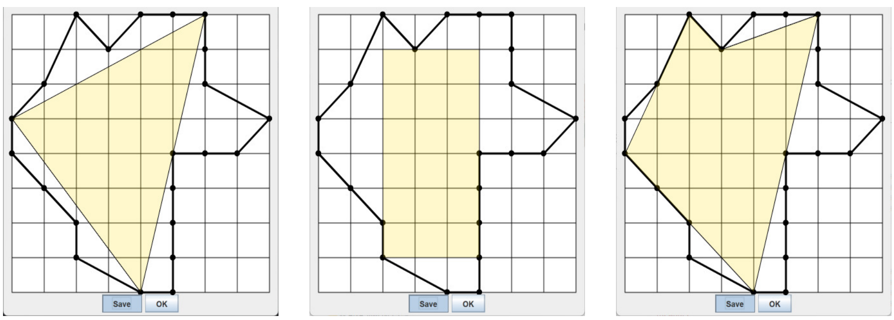


Figure 18. Task 4b. Rectangle, triangle, and simple pentagon of maximum area (PickSolver)

5.5. Observed difficulties and final reflections

The main difficulties identified were related to the use of the traditional method. Most of the errors arose first when counting the number of interior and boundary points, and later when applying the generalized version of Pick's theorem. Nevertheless, the combination of both traditional and digital methods proved beneficial. While the traditional method allowed for a deeper understanding of geometric concepts, the use of the software facilitated the verification of solutions, resulting in time savings throughout the process, as well as the acquisition of technological skills.

The activity based on a real-life situation, such as that proposed in Task 4, generated the greatest interest among the students. They reported that working on geometric problems with a clear and well-defined purpose significantly increased their willingness to remain engaged in the development of the task.

Collaboration among students facilitated the development of skills such as decision-making and communication, which provided greater confidence to the groups when facing problem-solving tasks. Some students also indicated that they understood the problems better when explaining them to their peers, reinforcing the previously stated idea that peer learning is a valuable pedagogical tool for deepening understanding and promoting greater engagement.

5.6. Comparison between the manual and digital methods

Table 3 presents a comparative summary between the traditional method (paper-and-pencil, PPB) and the digital method (PickSolver), based on the results and reflections of the student groups. Its construction takes into account aspects such as the complexity of the calculations required to solve each task, the level of coordination among group members, and the degree of student engagement, among others.

Both methods should be regarded as complementary rather than mutually exclusive; their combination enhances comprehension and creates a more flexible and motivating learning environment for students. PickSolver did not replace mathematical reasoning, but rather supported it, helping to make the learning process more dynamic.

Table 3. Comparative summary of manual and digital methods in geometric tasks

Session	Method	Observations
S_1	**PPB Method**	Correct use of Pick's theorem Difficulty identifying optimal triangle visually Promoted geometric reasoning
	PickSolver	Immediate visualization of the optimal triangle Easy experimentation with configurations Provided assurance in manual solutions
S_2	**PPB Method**	Frequent errors counting points in holes Challenging use of generalized formula Required strong group coordination
	PickSolver	Clear display and automatic hole handling Helped detect and correct errors
S_3	**PPB Method**	Group coordination essential Requires sustained attention
	PickSolver	Accurate display Reduced effort
S_4	**PPB Method**	Reasonable but not always optimal proposals Manual method limited for optimization Fostered critical thinking and discussion
	PickSolver	Automatic generation of optimal solutions Clear and immediate visual display Increased motivation and engagement

6. Conclusions

This paper demonstrates, through four tasks, that integrating traditional methods with digital tools such as PickSolver is both feasible and highly beneficial for geometry teaching and learning in the classroom. While the traditional method fosters the development of geometric reasoning, PickSolver enhances the visual understanding of concepts.

As a result, the software has proven to be a valuable educational tool. It can handle polygons with holes of any number of sides and arrive at solutions that would be very difficult to achieve manually. This allows students to experiment, make mistakes, and improve upon the solutions they previously found through traditional methods.

The work carried out by each of the four student groups, particularly that of Group 2, indicates that collaborative learning is effective in teaching the resolution of geometric problems. Students not only improved their computational skills through the application of Pick's theorem, but also made progress in their communication abilities. Furthermore, the use of PickSolver provided an opportunity to refine their solutions collaboratively, which contributed to an overall positive outcome.

Code Availability

The complete software package is openly accessible via the Zenodo repository (Molano 2025). The package includes the `.jar` and `.exe` files required to run the PickSolver application, along with a user guide and several example problems illustrating the software's functionalities. The software is compatible with Windows systems and requires Java SE Development Kit 24.0.1 or later. The authors assume full responsibility for the software and its use in the context of this study.

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