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DESIGN AND IMPLEMENTATION OF AUTO-VERIFYING GEOMETRY APPLETS FOR DIGITAL LEARNING

Peter Vajo, Ágnes Bazso, Dániel Tóth
Department of Mathematics, J. Selye University (Slovakia)

Abstract. Dynamic Geometry Systems have advanced considerably in recent years, yet the lack of open-source solutions that can automatically verify answers and provide feedback on geometric construction exercises still limits their wider use in education. This challenge arises from the inherent difficulty of assessing complex geometric constructions with computational methods. GeoTry was designed to address this need as an interactive digital learning resource developed with GeoGebra. It supports both teachers and learners by automatically verifying geometric construction exercises and offering structured guidance based on Pólya's problem-solving approach. We investigate how automatic verification can be meaningfully embedded in geometric construction exercises, focusing on the underlying design principles and technical considerations.

Keywords: GeoGebra; geometric construction problems; interactive learning; automatic verification; scaffolding; feedback

1. Introduction

The rapid development of educational technologies has resulted in a growing demand for dynamic mathematics software in mathematics classes (Stols and Kriek, 2011; Zengin and Tatar, 2017). The implementation of information and communication technologies (ICT) fosters a student-centered educational environment, increases student engagement, and promotes conceptual understanding (Bhagat et al., 2015; Hegedus et al., 2015). ICT tools serve a visualization function that enhances learning by boosting motivation and focused attention while providing immediate feedback that facilitates understanding the connections between abstract concepts and their practical applications (Jägerskog, 2020). Recent research demonstrates that technology-enhanced learning environments, particularly those integrating dynamic geometry software with web-based platforms, significantly improve students' conceptual understanding (Maizora et al., 2025). Multiple studies indicate that interactive applets and e-textbook learning approaches improve students' educational experiences by fostering achievement, critical thinking, engagement, and motivation (Sun and Pan, 2021; Chuang et al., 2023; Liu et al., 2020; Kartal and Çınar, 2024; Yildiz and Arpaci, 2024). Geometric thinking is fundamental to mathematical reasoning and problem-solving skills. Assessment challenges, however, often marginalize geometry education, as evidenced by the declining success rate in solving geometry problems in national examinations (Csiba and Vajo, 2024), which indicates a negatively developing disparity between geometry and other mathematical topics. This trend may lead to a long-term narrowing of students' cognitive abilities (Pavlovičová and Bočková, 2021).

In response to these challenges, we created GeoTry, a digital collection of exercises offering didactically refined, motivating exercises that enhance geometric thinking. The exercises support independent learning, reduce teachers' workload, and facilitate digital modernization of geometry education by using GeoGebra dynamic geometry software as interactive applets with automatic verification in geometric construction topics for secondary and tertiary education. The applets utilize dynamic visualizations and interactive exercises to engage students, while immediate feedback and successful problem-solving promote satisfaction and create a positive motivational cycle (Radović et al., 2018).

Based on a survey of mathematics teachers in secondary grammar schools with Hungarian as the language of instruction in the Slovak Republic, we identified three challenging areas in plane geometry linked to geometric construction problems: triangle construction problems, congruence transformations, and similarity transformations. These topics are incorporated into university mathematics teacher training curricula in Slovakia, enabling the materials to be utilized in tertiary education.

2. Pólya's problem-solving model

Pólya's (Polya, 1957) problem-solving model provides a structured framework particularly suited for addressing the complexities of geometric construction. Each construction step is guided by logical relationships between successive actions and the geometric properties inherent to the problem. Central to Pólya's model is the formulation of a coherent plan that connects given information with the desired outcome through systematic reasoning.

In his work, How to Solve It, Pólya outlines a four-step process that begins with understanding the problem — identifying all relevant data, objectives, and unknowns. This stage establishes the foundation for an effective plan that links known information to what must be determined. It involves applying geometric principles, sketching figures to visualize relationships, and invoking appropriate theorems to clarify logical connections (Polya, 1957). Once the plan is devised, it is implemented through precise construction steps that adhere to the established logical sequence. The final stage entails evaluating the solution for accuracy, coherence, and completeness, often leading to insights about alternative methods and deeper geometric understanding.

Modern educational resources continue to draw on Pólya's framework. For example, the textbook Matematika (Gerőcs and Vancsó, 2023) and digital platforms such as GeoTry — an open-source tool employing GeoGebra applets — apply Pólya's model to scaffold students' problem-solving processes in ways aligned with the learning principles described by Rogoff (1990).

In essence, Pólya's systematic methodology remains foundational in geometric problem solving, offering educators and learners a clear path from analysis to reflection. When combined with interactive technologies, it not only reinforces logical reasoning but also cultivates enduring problem-solving skills.

3. Some aspects of previous research

GeoGebra is a free, pedagogically designed platform that combines tools for geometry, algebra, statistics, calculus, spreadsheets, and graphical representation into an intuitive, user-centric interface (Jancheski and Jancheska, 2019). It facilitates swift hypothesis testing, validation of mathematical theorems, and investigation of critical aspects in geometric objects and functions, along with intuitive demonstrations of formal proofs. Due to its modular design, pre-existing educational resources can be tailored to suit various age groups and curriculum specifications: specific simulations can be either simplified or elaborated at different levels, while parameterization offers the versatility to create numerous variations of a single exercise. GeoGebra's dynamic, interactive interface and immediate visual feedback boost student interest and promote deeper comprehension; research substantiates its efficacy in problem-solving and modeling physical processes (Hohenwarter et al., 2007). GeoGebra functions as an instructional platform that transcends simple visualization; interactive materials, collaborative capabilities, and a straightforward interface render it a valuable conceptual and pedagogical resource (Bu and Schoen, 2011). It is useful at all educational levels for students of diverse abilities, and it is also efficiently utilized in the teaching of other STEM disciplines.

Several studies have demonstrated the effectiveness of GeoGebra in mathematics education. Owusu et al. (2023) found that using GeoGebra improved university students' understanding of polar coordinates, enhanced their achievement, and fostered more positive attitudes toward mathematics. They advocate for integrating such tools into tertiary curricula, especially in conceptually challenging topics. Similarly, Bedada and Machaba (2022) reported that GeoGebra-assisted teaching significantly improved STEM students' performance in trigonometric functions by enhancing their ability to connect representations and interpret graphs. The study highlights GeoGebra's role in promoting active, student-centered learning environments.

Research concerning the development of geometric construction problems with automatic verification and their incorporation into a learning management system that monitors student progress is notably limited. GeoTest, created by Gergelitsová and Holan (2012), is among the most significant experiments in recent years. This custom-developed system requires registration, features automatic assessment, integrates GeoGebra into a website, and is predominantly utilized in the Czech Republic (Gergelitsová and Holan, 2016). The website matek.ujs.sk, created and operated by the Department of Mathematics at J. Selye University, is also noteworthy, as highlighted by Csiba and Vajo (2024). Upon registration and user creation, teachers can monitor students' progress, concentrating on mathematics subjects for the second stage of primary education. GeoGebra assignments are integrated into Moodle via a plugin, necessitating minimal IT or programming expertise from the mathematics teacher.

4. Learning geometric concepts in GeoTry

GeoTry is a free-of-charge collection of exercises built with GeoGebra that features randomized, interactive geometric construction exercises with automatic verification, incorporating Pólya's problem-solving steps and instructional scaffolding. GeoTry is organized into three thematic units:

- triangle construction problems,
- congruence transformations,
- and similarity transformations.

All GeoTry applets are dynamically randomized, preventing students from relying on the memorization of any specific figure and instead promoting the generalization of patterns. This feature offers nearly limitless practice opportunities and develops the abstract reasoning skills crucial for transferable mathematical cognition. We have made GeoTry available as an interactive GeoGebra Book. A GeoGebra Book is essentially a collection of instructional units divided into chapters, each of which pairs concise theoretical explanations with interactive GeoGebra applets. Although we explored the option of developing an independent website, we ultimately selected the GeoGebra platform because of its comprehensive authoring tools and, in particular, its real-time student-monitoring capabilities, which allow teachers to track learners' progress and interactions live.

GeoTry consists of two layers: the presentation layer, which encompasses the user interface, and the system layer, which contains the background operations that guarantee data randomization in the geometric construction problems and accurate verification of solvers' solutions by JavaScript.

4.1. The presentation layer

Figure 1 illustrates the solver's perspective when opening a selected GeoTry applet. The Toolbar, located at the top of the applet, encompasses all the tools necessary for constructing the solution, with the components configurable to a specific selection. This means the teacher can modify the toolbar to ensure that only Euclidean tools are accessible to the solver. The applets feature three buttons that support the GeoTry concept: the Refresh button, which randomly alters the given data; the Help button, which presents a scaffold (a hint, or a visual or textual help); and the Verification button, which offers feedback on the accuracy of the solver's constructed solution.

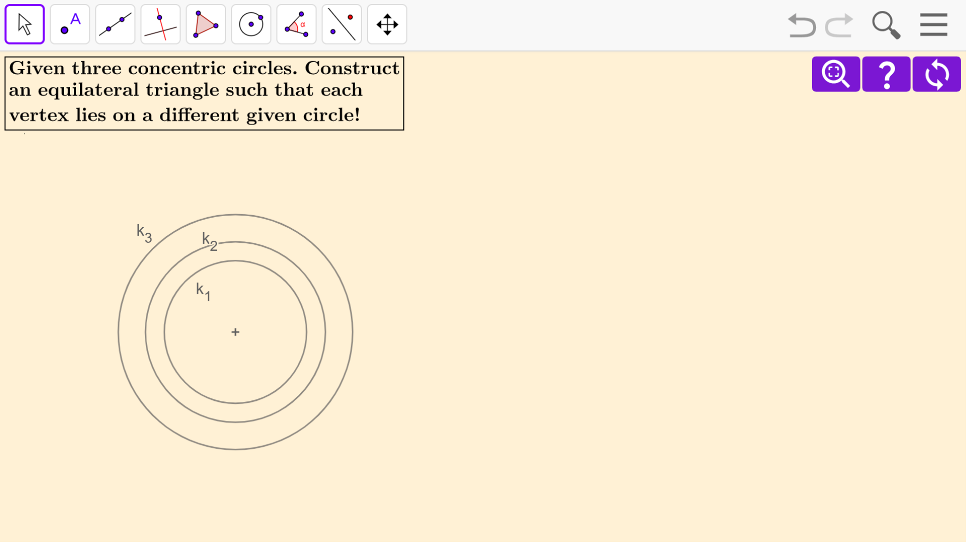

Figure 1. The student interface of a selected GeoTry exercise

The dimensions of GeoTry's applets are set to ensure that the preview displayed, even on a compact office laptop, tablet, or smartphone, encompasses the applet and all its components in their entirety. This is especially pertinent in the context of Apollonius problems, which include several circles with significant differences in radius.

GeoTry implements scaffolding via task-specific hints, utilizing visual representations and highlighting key geometric properties to clarify theoretical foundations and guide effective problem-solving. The scaffolds aim to clarify the theoretical foundations of geometric constructions, thereby facilitating problem-solving and promoting conceptual understanding.

As an example, an exercise from the triangle construction unit (Fig. 2) involves constructing a right-angled triangle given its altitude and the median of its hypotenuse. The solution relies on the notion of Thales's theorem, which states that the median of the hypotenuse of a right-angled triangle is the circumradius. The 'Help' button activates a scaffold by presenting Thales's theorem and displaying an interactive figure that visually connects the theorem to the given data, the median as circumradius, and the altitude, thus bridging theory with practical construction.

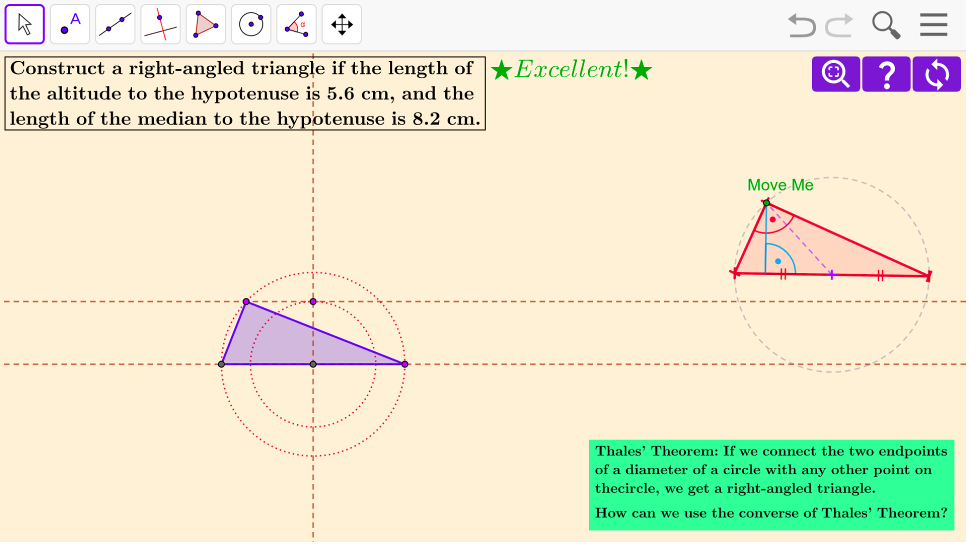

Figure 2. A successfully constructed solution with hints

Furthermore, exercises advance from basic exercises like Heron's problem to more intricate challenges such as Apollonius' problems, reflecting the educational idea of gradually eliminating support as learners develop confidence and proficiency.

GeoTry incorporates an interactive feedback system designed to enhance learning through real-time verification and guided problem-solving. Users can validate their solutions at any stage by clicking the Verification button (with the magnifying glass icon), which triggers immediate textual feedback. This feedback is communicated through both color-coded indicators and descriptive content, allowing students to identify errors and correct their constructions.

By combining adaptive exploration with structured guidance, GeoTry harmonizes intuitive discovery and formal mathematical reasoning. This dual approach positions the platform as a versatile educational tool that fosters student-centered exploration while cultivating systematic discourse and deeper conceptual understanding.

4.2. The system layer — Randomization and verification

Upon the launch of an applet by a user, given data — such as side lengths, coordinates, and angles — are algorithmically randomized, guaranteeing that every applet's launch offers a different challenge.

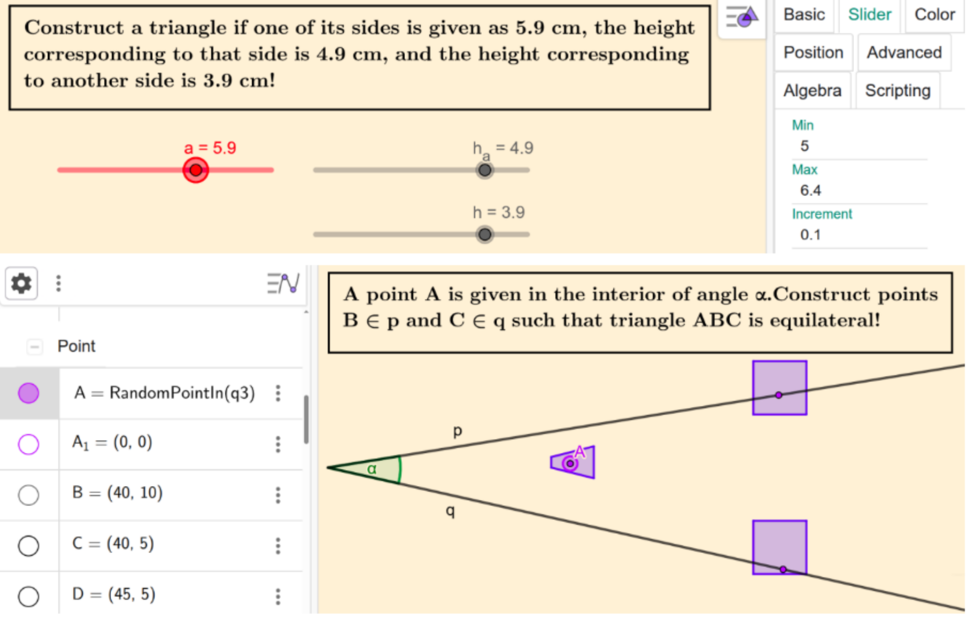

Figure 3. Randomization options of GeoTry

The creation of randomized exercises requires systematic ways for producing changeable geometric elements, such as lines, points, circles, and other objects, together with their textual descriptions. A successful method entails utilizing randomized slider controls (see Fig. 3), which dynamically modify numerical parameters such as segment lengths (triangle side lengths) or radius lengths of circles. A similar approach utilizes object-bound randomization, wherein points are systematically created inside specified geometric areas (see Fig. 3).

JavaScript is employed to create applets that continuously monitor students' work (the constructed objects) and, after using the Verification button, instantly inform them about the correctness of their solution or indicate the necessity for additional changes, as in the case of problems with more than one correct, non-congruent solution. JavaScript-based GeoGebra applets can identify the addition, removal, or alteration of geometric objects, providing immediate feedback accordingly.

In the case of geometric constructions, the general task is to construct a specific geometric object. The correct solution, depending upon random variables, has been pre-constructed, and its visibility is disabled. Additionally, two text objects were established for feedback — „passFeedbackText“ for a successful solution and „failFeedbackText“ for an unsuccessful one — both with their visibility initially disabled. The button that initiates the evaluation is designated as „evalButton“. The included scripts were incorporated into the applet, as outlined, by embedding them within the Global JavaScript field available in the Scripting tab of the object's properties.

```
function ggbOnInit() {
    ggbApplet.registerAddListener("eventListener");
    ggbApplet.registerClickListener("eventListener");
}
function eventListener(ggbElement) {
    if (ggbElement.includes("finished")) {return;}
    var isFinished = ggbApplet.getValue("finished") == 1;
    if (!isFinished) {
        var cmd = "finished = AreCongruent(" + ggbElement + ",
            solution)";
        ggbApplet.evalCommand(cmd);
    }
    var showPassFeedback = false;
    var showFailFeedback = false;
    if (ggbElement == "evalButton") {
        showPassFeedback = isFinished;
        showFailFeedback = !isFinished;
    }
    ggbApplet.setVisible("passFeedbackText", showPassFeedback);
    ggbApplet.setVisible("failFeedbackText", showFailFeedback);
}
```

Listing 1. GeoGebra event listener for task evaluation

The JavaScript code's line
`var cmd = "finished = AreCongruent(" + obj + ",solution)";`
compares all newly constructed objects with the hidden, predefined solution. If the two items are congruent, the system displays a message confirming a

successful answer; otherwise, feedback regarding an inaccurate or incomplete solution is revealed to the solver by the use of the Verification button.

In exercise design, it is essential to first determine the appropriate type of geometric comparison and place the corresponding GeoGebra comparison function into the script. For example, when comparing lines, `AreCongruent` is not used—since any two lines are congruent under translation — but instead `AreEqual` is employed to test exact coincidence. In contrast, when comparing triangles or other polygons, the focus is solely on congruence (shape and size), independent of position or orientation, and thus `AreCongruent` is employed. GeoGebra provides the following comparison functions:

- `AreCollinear`: tests whether points lie on a common line,
- `AreConcurrent`: tests whether lines meet at a single point,
- `AreConcyclic`: tests whether points lie on the same circle,
- `AreCongruent`: tests for congruence (invariant under rotation and translation),
- `AreEqual`: tests for exact coincidence,
- `AreParallel`,
- `ArePerpendicular` (GeoGebra Team, 2025).

The == operator is avoided, as it performs a purely syntactic comparison without considering object types. As a result, it may incorrectly equate different types of objects, for example, the numerical value 1, a unit-length segment, and a unit-area square. However, one can design an exercise that requires precisely this type of behavior.

The GeoGebra JavaScript API provides real-time monitoring of student interactions and facilitates immediate feedback. Upon loading the construction, the `ggbOnInit` function may be employed to establish listener methods that respond to user events, enabling the applet to react to each new object created and every click made. For instance, when a student adds a new point to the construction interface, the function registered through the `registerAddListener` method receives the object's name and runs the corresponding logic.

The `registerClickListener` function can be utilized to monitor clicks on any graphical element inside the applets. The name of the clicked object is transmitted to the invoked JavaScript function, which can, for example, evaluate the selection of the point and display feedback based on the result.

The `evalCommand` method allows the program to run GeoGebra commands, including the concurrent construction or alteration of many objects using loop structures. The `setVisible` function enables the dynamic concealment or display of graphical elements, ensuring that feedback labels such as „Correct!“ or „Incorrect!“ are shown only when warranted by the evaluation of the student's actions.

The integration of these methodologies fosters a versatile, dynamic learning environment that surpasses the functionalities of GeoGebra's built-in scripting language. As Banchev (2024) notes, while construction languages describe geometric dependencies, features like conditional execution and loops – provided here by JavaScript – are essential for sophisticated, algorithmic interactivity. These structures allow students to manage complex logical conditions and receive immediate feedback, enabling them to directly observe the consequences of both correct and incorrect solutions, thereby deepening conceptual understanding.

5. Usability in education

For students, GeoTry is an interactive tool that offers guidance and feedback, along with a wide variety of geometric construction problems. For mathematics teachers, it may also function as a collection of exercises, as well as a visualization and assessment tool at both secondary and tertiary levels. The subsequent section outlines the potential applications of GeoTry for mathematics teachers.

We have developed comprehensive solution guides with construction steps for each GeoGebra applet (see Figure 4). These guidelines outlining the construction steps, derived from the original applets, offer detailed explanations aimed at enhancing teachers' preparation while remaining available as reference materials for students, at the teacher's discretion.

A slider is located in the top-right corner, enabling teachers or students to manage and navigate the construction processes. The applet includes a formal description of the construction steps, which incrementally becomes visible with each stage. This dual methodology — visual engagement combined with textual instruction — promotes clarity and facilitates a comprehensive understanding of both the practical and theoretical dimensions of the work.

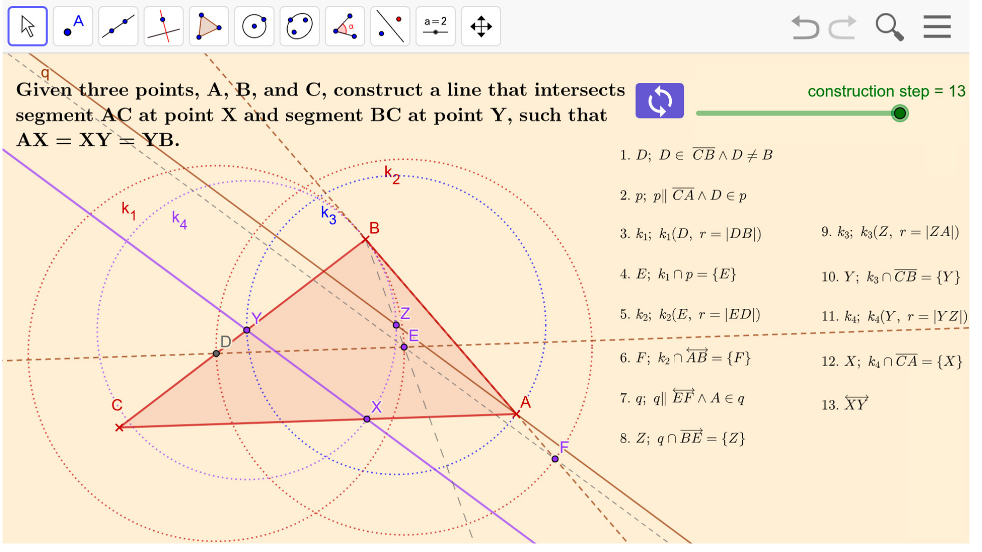

Figure 4. The solution steps to a selected exercise

By using GeoGebra Classroom, teachers can see their students' progress in a list-like layout where each applet window displays what the students' current construction is in real time, as shown in Figure 5. This offers the opportunity to monitor the students' work (homework assignments, distance learning and potential testing) and assess it by using the Verification button, saving precious time compared to traditional verification methods, and allowing teachers to maintain oversight and support student progress more effectively.

Figure 5. The GeoTry teacher interface in GeoGebra classroom

6. Conclusion

The integration of automatic verification within Dynamic Geometry Systems offers significant pedagogical advantages. Our open-source platform, GeoTry, addresses this need by embedding a comprehensive, automated verification system within the GeoGebra environment. It provides an adaptive learning resource that supports both autonomous student exploration through "what if?" scenarios and structured classroom instruction. The platform's pedagogical design, incorporating Pólya's problem-solving model and task randomization, fosters deep conceptual understanding over procedural memorization.

For educators, GeoTry serves as a standardized resource for visualization and practice, a particularly valuable contribution in educational contexts lacking standardized textbooks. It reduces the workload on teachers while offering powerful tools for real-time monitoring. The current implementation includes over 50 interactive applets in Hungarian and English, with an open-source framework that permits further linguistic and content expansion. The complete collection of these interactive resources is available at `https://www.geogebra.org/m/mp5xpzqy`. While this study establishes the technical and pedagogical foundation, future research must prioritize empirical validation through classroom studies and Data Science Research for testing and evaluating GeoTry's performance and effectiveness and to measure its impact on student achievement and problem-solving skills.

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✉ **Peter Vajo**
ORCID iD: 0009-0009-0790-4743
Department of Mathematics
J. Selye University
Bratislavská cesta 3322
Komárno, Slovakia
E-mail: `petervajo4@gmail.com`

✉ **Ágnes Bazso**
ORCID iD: 0009-0002-4978-8233
Department of Mathematics
J. Selye University
Bratislavská cesta 3322
Komárno, Slovakia
E-mail: `bartal.agnes@gmail.com`

✉ **Dániel Tóth**
ORCID iD: 0009-0007-5712-8426
Department of Mathematics
J. Selye University
Bratislavská cesta 3322
Komárno, Slovakia
E-mail: `tothd@ujs.sk`